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(54) **METHODS AND APPARATUS FOR DATA TRANSFER OPTIMIZATION**

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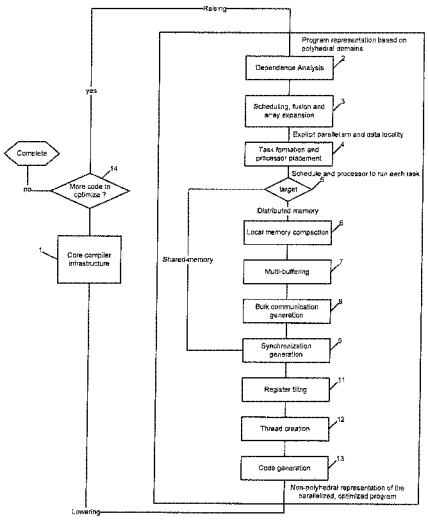
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(57) **ABSTRACT**

Methods, apparatus and computer software product for optimization of data transfer between two memories includes determining access to master data stored in one memory and/or to local data stored in another memory such that either or both of the size of total data transferred and the number of data transfers required to transfer the total data can be minimized. The master and/or local accesses are based on, at least in part, respective structures of the master and local data.

22 Claims, 9 Drawing Sheets



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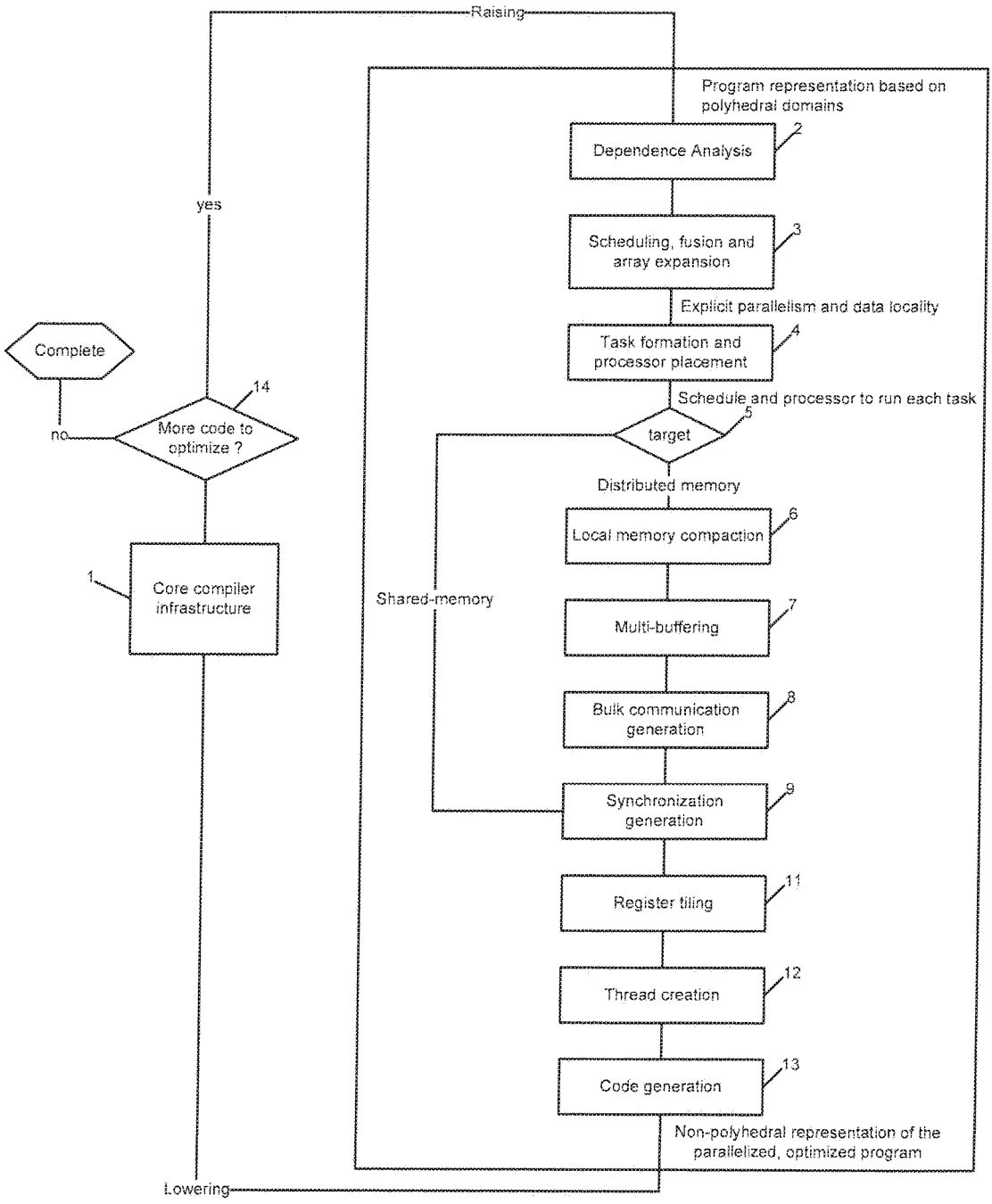

Raising
Program representation based on polyhedral domains
Dependence Analysis
2
Scheduling, fusion and array expansion
3
Explicit parallelism and data locality
Task formation and processor placement
4
Schedule and processor to run each task
target
5
Distributed memory
Local memory compaction
6
Shared-memory
Multi-buffering
7
Bulk communication generation
8
Synchronization generation
9
Register tiling
11
Thread creation
12
Code generation
13
Non-polyhedral representation of the parallelized, optimized program
yes
Complete
no
More code to optimize ?
14
Core compiler infrastructure
1
Lowering

FIG. 1

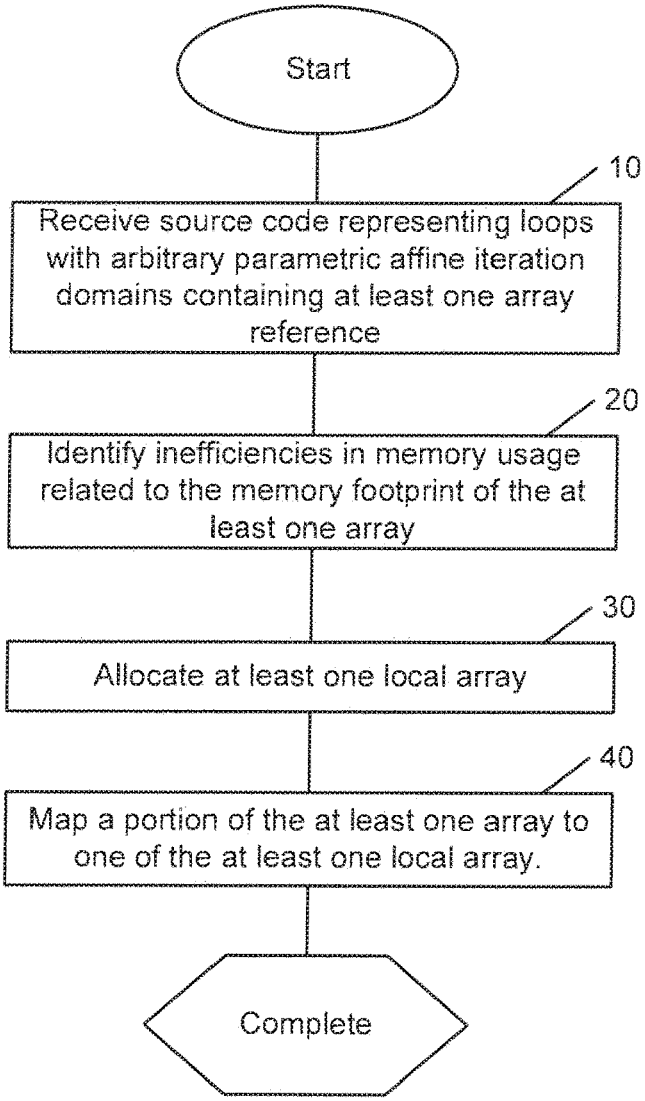

Start
10
Receive source code representing loops with arbitrary parametric affine iteration domains containing at least one array reference
20
Identify inefficiencies in memory usage related to the memory footprint of the at least one array
30
Allocate at least one local array
40
Map a portion of the at least one array to one of the at least one local array.
Complete

FIG. 2

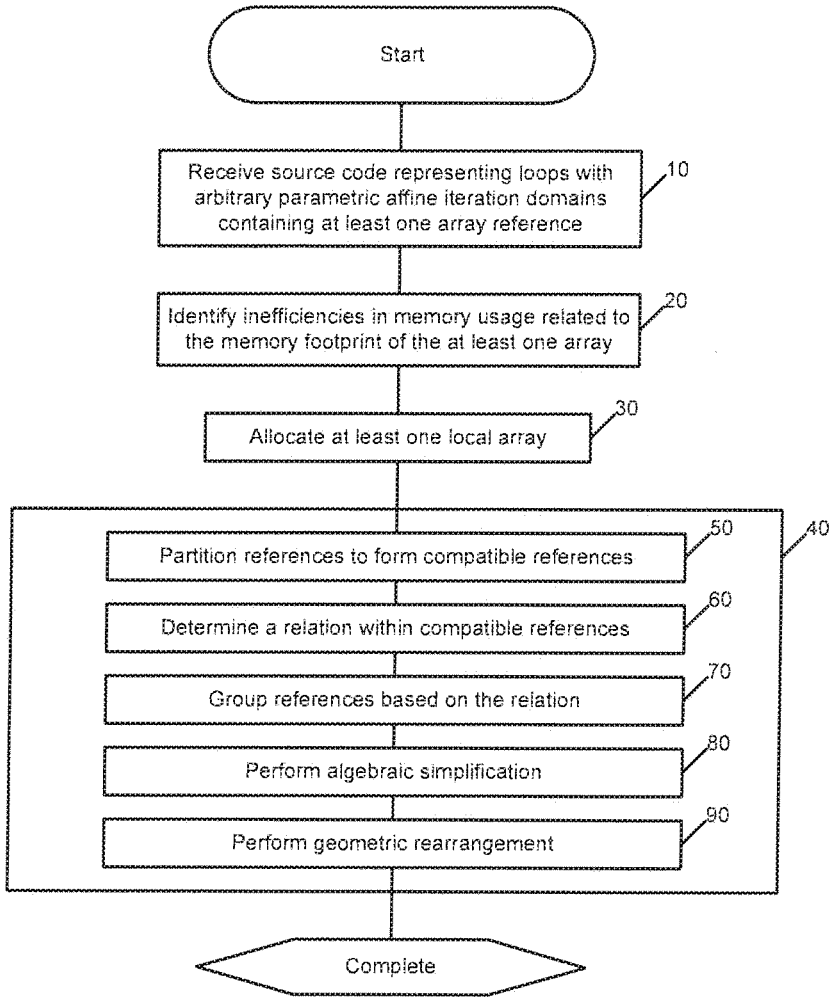

Start
Receive source code representing loops with arbitrary parametric affine iteration domains containing at least one array reference
10
Identify inefficiencies in memory usage related to the memory footprint of the at least one array
20
Allocate at least one local array
30
40
Partition references to form compatible references
50
Determine a relation within compatible references
60
Group references based on the relation
70
Perform algebraic simplification
80
Perform geometric rearrangement
90
Complete

FIG. 3

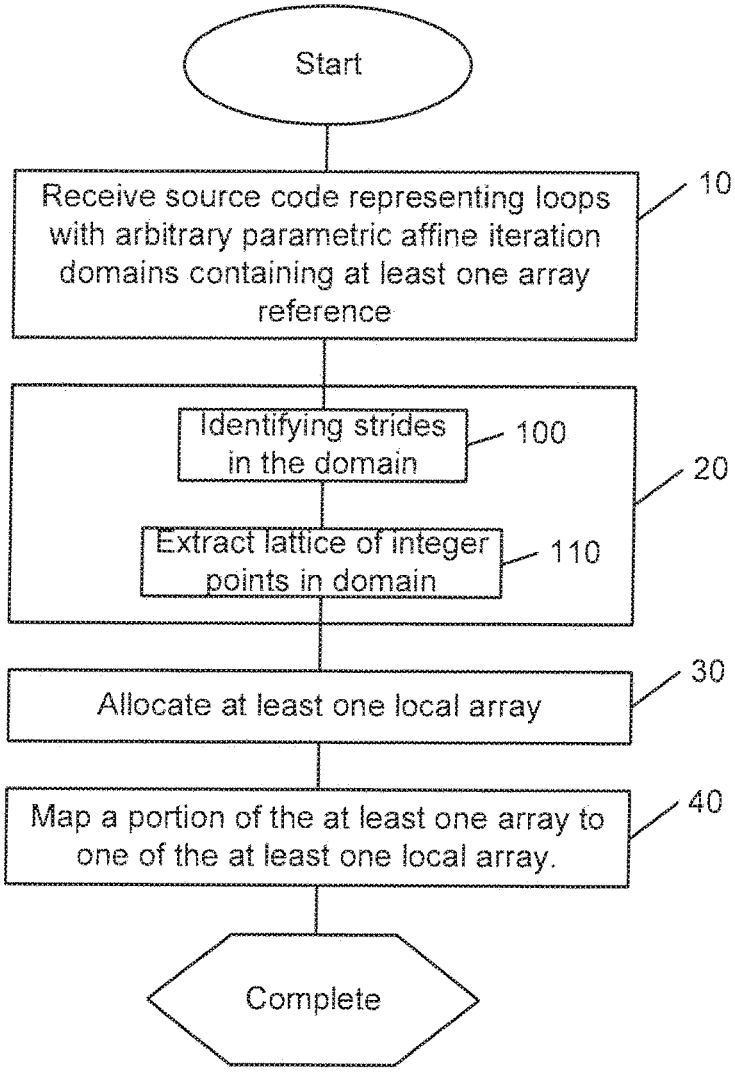

Start
Receive source code representing loops with arbitrary parametric affine iteration domains containing at least one array reference
10
Identifying strides in the domain
100
Extract lattice of integer points in domain
110
20
Allocate at least one local array
30
Map a portion of the at least one array to one of the at least one local array.
40
Complete

FIG. 4

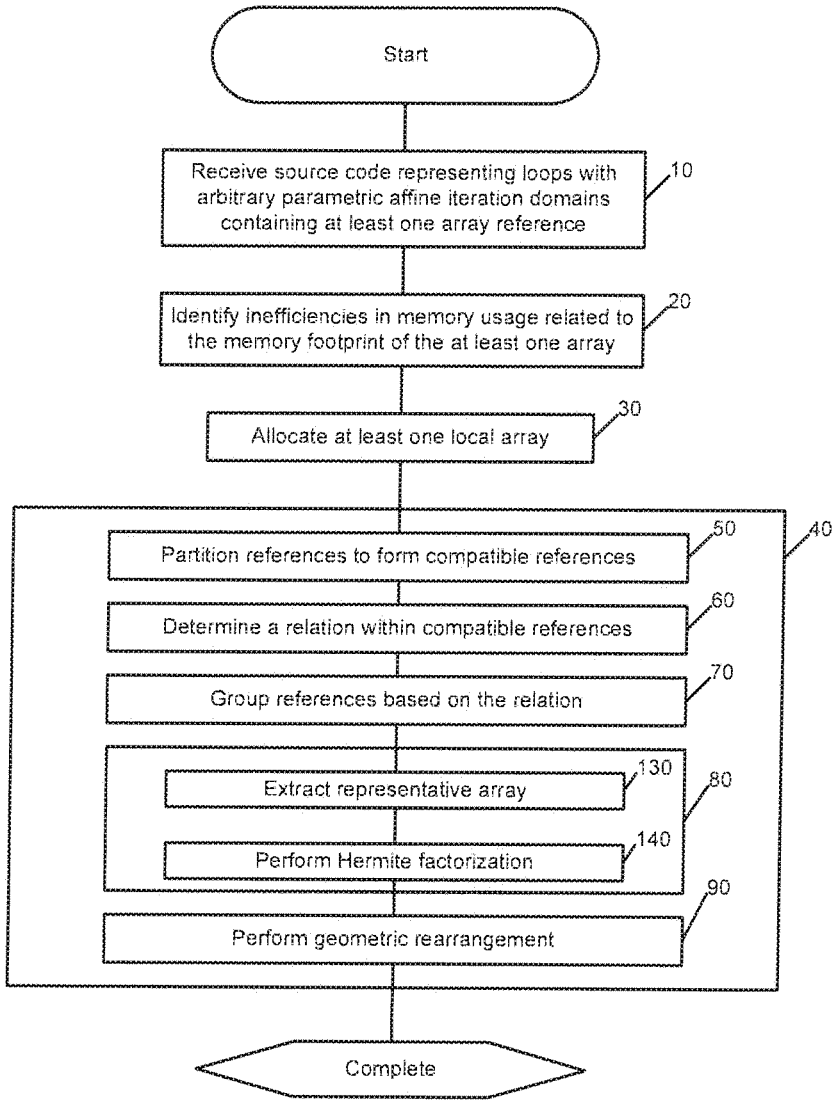

Start
Receive source code representing loops with arbitrary parametric affine iteration domains containing at least one array reference
10
Identify inefficiencies in memory usage related to the memory footprint of the at least one array
20
Allocate at least one local array
30
40
Partition references to form compatible references
50
Determine a relation within compatible references
60
Group references based on the relation
70
Extract representative array
130
Perform Hermite factorization
140
80
Perform geometric rearrangement
90
Complete

FIG. 5

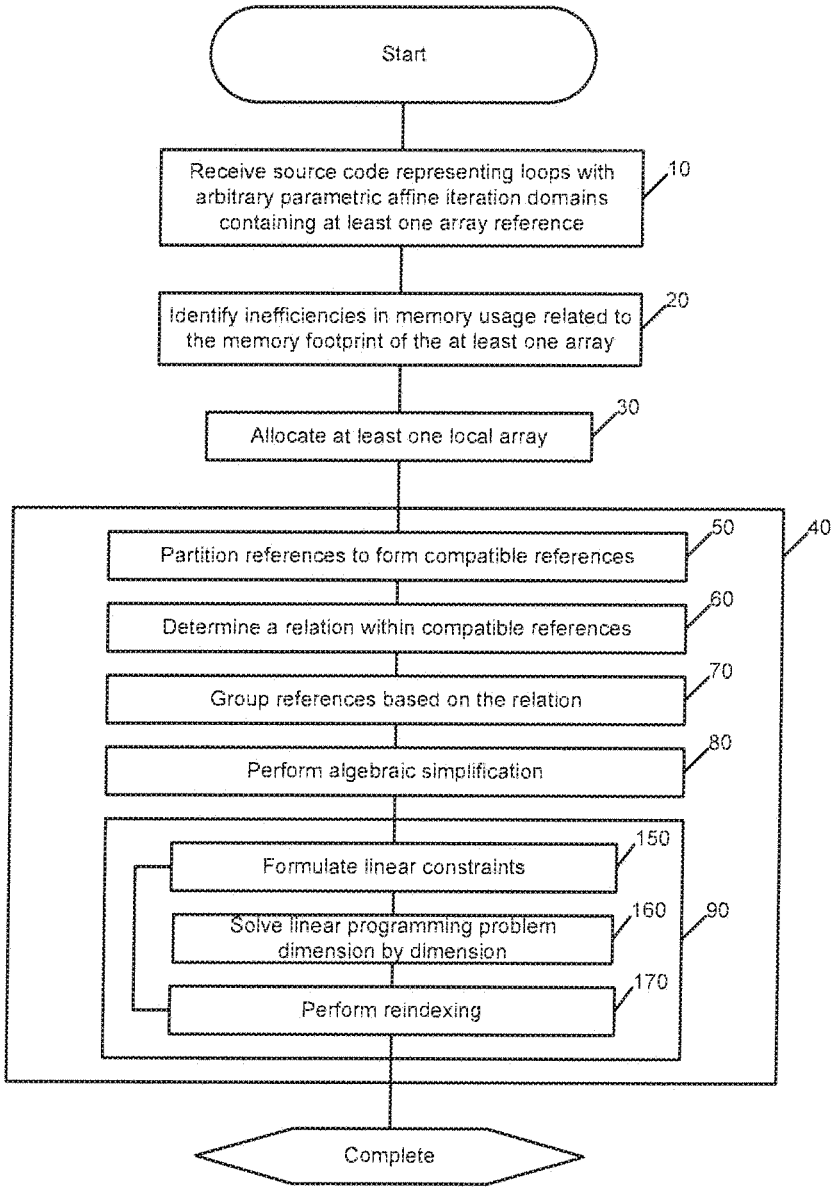

Start
Receive source code representing loops with arbitrary parametric affine iteration domains containing at least one array reference
10
Identify inefficiencies in memory usage related to the memory footprint of the at least one array
20
Allocate at least one local array
30
40
Partition references to form compatible references
50
Determine a relation within compatible references
60
Group references based on the relation
70
Perform algebraic simplification
80
Formulate linear constraints
150
Solve linear programming problem dimension by dimension
160
90
Perform reindexing
170
Complete

FIG. 6

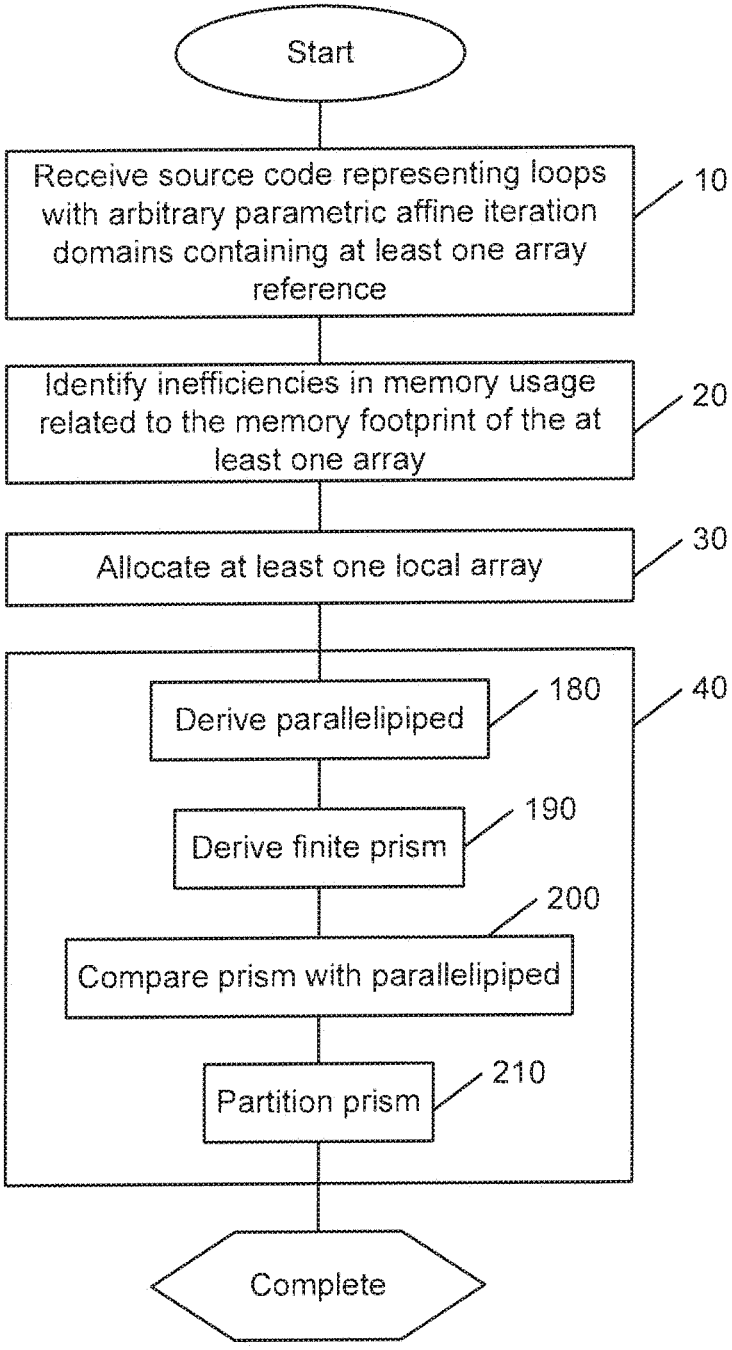

Start
Receive source code representing loops with arbitrary parametric affine iteration domains containing at least one array reference
10
Identify inefficiencies in memory usage related to the memory footprint of the at least one array
20
Allocate at least one local array
30
40
Derive parallelipiped
180
Derive finite prism
190
Compare prism with parallelipiped
200
Partition prism
210
Complete

FIG. 7

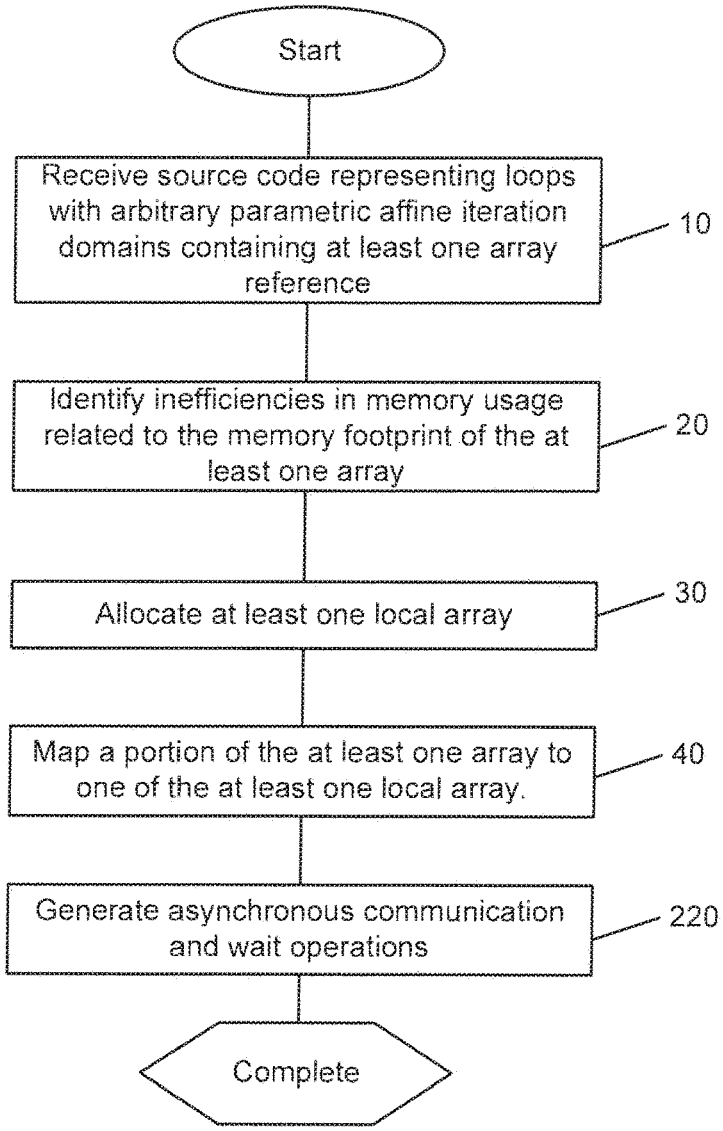
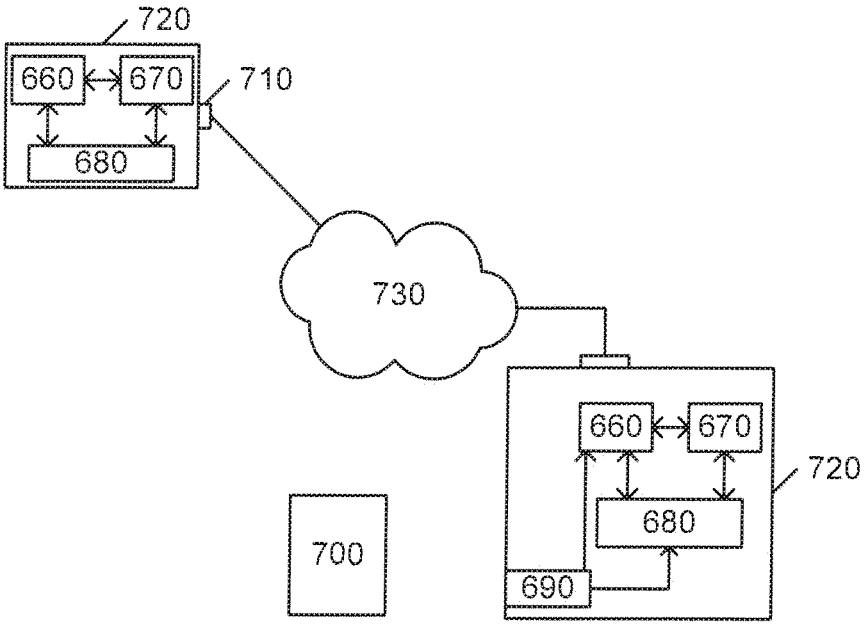

Start
Receive source code representing loops with arbitrary parametric affine iteration domains containing at least one array reference
10
Identify inefficiencies in memory usage related to the memory footprint of the at least one array
20
Allocate at least one local array
30
Map a portion of the at least one array to one of the at least one local array.
40
Generate asynchronous communication and wait operations
220
Complete

FIG. 8


720
660
670
710
680
730
660
670
720
680
700
690

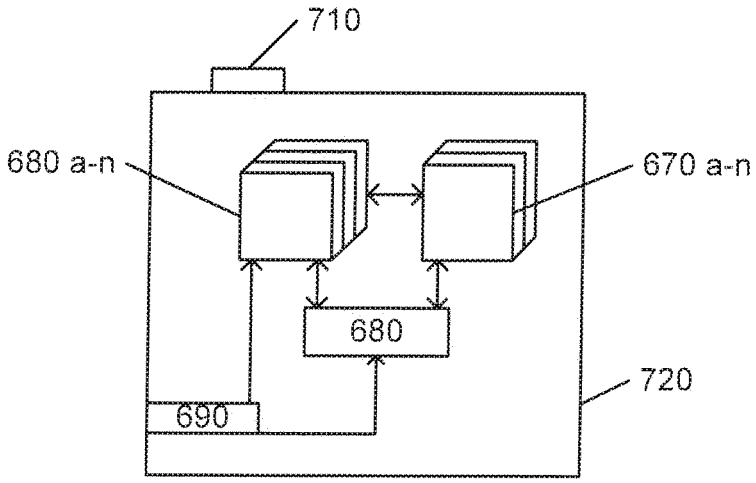

710
680 a-n
670 a-n
680
720
690

FIG. 9

METHODS AND APPARATUS FOR DATA TRANSFER OPTIMIZATION

CROSS REFERENCE TO RELATED APPLICATIONS

This application is a continuation of U.S. patent application Ser. No. 14/181,201, entitled "Methods and Apparatus For Data Transfer Optimization," filed Feb. 14, 2014, which is a continuation-in-part of U.S. patent application Ser. No. 12/365,780, entitled "Methods And Apparatus For Local Memory Compaction," filed on Feb. 4, 2009, which is related to and claims the benefit of priority to U.S. Provisional Application Ser. No. 61/065,294, entitled "SYSTEM, APPARATUS, AND METHODS FOR SOURCE CODE COMPILATION", filed Feb. 8, 2008, each of which is incorporated herein by reference in its entirety.

GOVERNMENT INTEREST

Portions of this invention were made with U.S. Government support under contract/instrument DARPA F03602-03-C-0033 with the U.S. Air Force Research Laboratory and DARPA. The U.S. Government has certain rights.

FIELD OF THE INVENTION

The present invention generally concerns computer programming. More particularly, the invention concerns a system, methods, and apparatus for source code compilation.

BACKGROUND OF THE INVENTION

The progression of the computer industry in recent years has illustrated the need for more complex processor architectures capable of processing large volumes of data and executing increasingly complex software. A number of systems resort to multiple processing cores on a single processor. Other systems include multiple processors in a single computing device. Additionally, many of these systems utilize multiple threads per processing core. One limitation that these architectures experience is that the current commercially available compilers cannot efficiently take advantage of the increase of computational resources.

In the software design and implementation process, compilers are responsible for translating the abstract operational semantics of the source program into a form that makes efficient use of a highly complex heterogeneous machine. Multiple architectural phenomena occur and interact simultaneously; this requires the optimizer to combine multiple program transformations. For instance, there is often a trade-off between exploiting parallelism and exploiting locality to reduce the "memory wall", i.e., the ever widening disparity between memory bandwidth and the frequency of processors. Indeed, the speed and bandwidth of the memory subsystems are a performance bottleneck for the vast majority of computers, including single-core computers. Since traditional program optimization problems are associated with huge and unstructured search spaces, this combinational task is poorly achieved by current compilers, resulting in poor scalability of the compilation process and disappointing sustained performance of the supposedly optimized program.

Even when programming models are explicitly parallel (threads, data parallelism, vectors), they usually rely on advanced compiler technology to relieve the programmer from scheduling and mapping the application to computational cores, and from understanding the memory model and communication details. Even provided with enough static information and code annotations (OpenMP directives, pointer aliasing, separate compilation assumptions), traditional compilers have a hard time exploring the huge and unstructured search space associated with the mapping and optimization challenges. Indeed, the task of the compiler can hardly be called "optimization" anymore, in the traditional meaning of reducing the performance penalty entailed by the level of abstraction of a higher-level language. Together with the run-time system (whether implemented in software or hardware), the compiler is responsible for most of the combinatorial code generation decisions to map the simplified and ideal operational semantics of the source program to a highly complex and heterogeneous target machine.

Generating efficient code for deep parallelism and deep memory hierarchies with complex and dynamic hardware components is a difficult task. The compiler (along with the run-time system) now has to take the burden of much smarter tasks, that only expert programmers would be able to carry. In order to exploit parallelism, the first necessary step is to compute a representation which models the producer/consumer relationships of a program as closely as possible. The power of an automatic optimizer or parallelizer greatly depends on its capacity to decide whether two portions of the program execution may be run one after another on the same processing element or on different processing elements, or at the same time ("in parallel"). Such knowledge is related to the task of dependence analysis which aims at precisely disambiguating memory references. One issue is to statically form a compact description of the dynamic properties of a program. This process is generally undecidable and approximations have to be made.

Once dependence analysis has been computed, a compiler performs program transformations to the code with respect to different, sometimes conflicting, performance criteria. Any program transformation must ultimately respect the dependence relations in order to guarantee the correct execution of the program. A class of transformations targeting the loop nests of a program (such as "DO" loops in the FORTRAN language, and "for" and "while" loops in languages derived from the C language) are known to account for the most compute intensive parts of many programs.

Traditional optimizing compilers perform syntactic transformations (transformations based on a representation that reflects the way the program source code text was written, such as the Abstract Syntax Tree), making the optimizations brittle since they are highly dependent on the way that the input program is written, as opposed to the more abstract representation of the program's execution offered by the polyhedral model. Moreover, syntactic transformations are not amenable to global optimizations, since the problem of optimally ordering elementary syntactic transformations is yet unsolved. Many interesting optimizations are also not available, such as fusion of loops with different bounds or imperfectly nested loop tiling.

In some situations, such as in high performance signal and image processing, the applications may primarily operate on "dense" matrices and arrays. This class of applications primarily consists of do-loops with loop bounds which are affine functions of outer indices and parameters, and array indexing functions which are affine functions of loop indices and parameters. Other classes of programs can be approximated to that class of programs.

One significant area of concern in these large scale systems is memory management. For example, in a program, a large multi-dimensional array may be allocated and

used to store data. This large block of data is typically stored in memory in contiguous memory cells. Certain operations on the array may not access all portions of the data. For example, in nested loops an outer loop may be indexed by the column of the array and an inner loop may be indexed by the rows of the array. In a situation where the loop operation only accesses a portion of the elements of the array, it would be inefficient to transfer the entire array to a processing element that is assigned the access task. Further, since portions of the array are not accessed, the loop indices may be rewritten for local access on a processing element.

There have been a number of approaches used to implement these program transformations. Typical goals of these approaches include reducing the memory size requirements to increase the amount of useful data in local memory and to reduce communication volumes. One such algorithm is described in U.S. Pat. No. 6,952,821 issued to Schreiber. Schreiber's method is applicable to non-parametric rectangular iteration spaces and employs the Lenstra-Lenstra-Lovasz (LLL) lattice basis reduction algorithm. Schreiber's methods are additionally incapable of addressing data with non-convex sets of accessed data.

Therefore a need exists for more efficient compiler architectures that optimize the compilation of source code.

SUMMARY OF THE INVENTION

The present invention provides a system, apparatus and methods for overcoming some of the difficulties presented above. Various embodiments of the present invention provide a method, apparatus, and computer software product for a class of automatic program transformations that reduce the memory size requirements by relocating and compressing the memory accesses of a program that includes loop nests with arbitrary affine indices. Exemplary methods extract a lattice of points within the iteration domain to handle iteration domains with strides, for which the values of loop counters differ by more than a unit for loop iterations executed consecutively. Other provided methods operate on programs that contain arbitrary affine array index functions; and in some instances where the program transformation handles arbitrarily complex data footprints.

An exemplary method includes receiving program source code containing loop nests with arbitrary parametric affine iteration domain containing at least one array. The method identifies inefficiencies in memory usage where the inefficiencies are related to access and the memory footprint of the arrays. The method further allocates at least one local array and maps a portion of the received arrays to one or more of the local arrays. The mapping reduces the memory size requirements and the memory footprint of the arrays.

A further embodiment provides a local memory compaction module that assists a processor in the optimization of source code. Other embodiments provide computing apparatus and computer software products that implement the described methods.

BRIEF DESCRIPTION OF THE DRAWINGS

Various embodiments of the present invention taught herein are illustrated by way of example, and not by way of limitation, in the figures of the accompanying drawings, in which:

FIG. **1** is an overview of an exemplary compiler architecture consistent with provided embodiments;

FIG. **2** illustrates the operational flow of one embodiment of a provided local memory compaction module;

FIG. **3** illustrates the operational flow of another provided local memory compaction module, in which array references are partitioned into groups and algebraic and geometric data re-indexing functions are computed;

FIG. **4** illustrates the operational flow of an additional local memory compaction module in which inefficiencies in memory usage are determined using lattices of integer points;

FIG. **5** illustrates the operational flow of an additional local memory compaction module for reducing the inefficiencies in local memory usage by extracting representative array references and producing re-indexing functions using Hermite factorizations;

FIG. **6** illustrates the operational flow of an additional local memory compaction module for computing data re-indexing functions by producing linear constraints and solving a series of linear programming problems.

FIG. **7** illustrates the operational flow of an additional local memory compaction module for computing data re-indexing functions by finding a prism of triangular base that encloses the accessed data set and reducing the memory requirements of the enclosed data region by transforming the data elements that lie within a subset of the prism of triangular base.

FIG. **8** illustrates the operational flow of an additional local memory compaction module using data re-indexing information to produce abstract communication commands and schedule computations and communications for the program in such a way that their executions overlap; and

FIG. **9** illustrates a computing apparatus and computer software product consistent with provided embodiments.

It will be recognized that some or all of the figures are schematic representations for purposes of illustration and do not necessarily depict the actual relative sizes or locations of the elements shown. The Figures are provided for the purpose of illustrating one or more embodiments of the invention with the explicit understanding that they will not be used to limit the scope or the meaning of the claims.

DETAILED DESCRIPTION OF THE INVENTION

In the following paragraphs, the present invention will be described in detail by way of example with reference to the attached drawings. While this invention is capable of embodiment in many different forms, there is shown in the drawings and will herein be described in detail specific embodiments, with the understanding that the present disclosure is to be considered as an example of the principles of the invention and not intended to limit the invention to the specific embodiments shown and described. That is, throughout this description, the embodiments and examples shown should be considered as exemplars, rather than as limitations on the present invention. Descriptions of well known components, methods and/or processing techniques are omitted so as to not unnecessarily obscure the invention. As used herein, the "present invention" refers to any one of the embodiments of the invention described herein, and any equivalents. Furthermore, reference to various feature(s) of the "present invention" throughout this document does not mean that all claimed embodiments or methods must include the referenced feature(s).

The trend of increasing the frequency at which processors perform computations seems to have come to an end. Power consumption and control complexity have reached such high levels that manufacturers are backing out of this design path. Current machines have evolved to multiprocessor architec-

tures on a chip with increasingly many cores per chip and multiple threads per core. This trend is expected to dramatically increase, reaching thousands of cores per chip in the next few years. Thus, modern computers increasingly need to exploit parallelism at different levels to provide sustained performance. On the other hand, parallel programming techniques have not evolved at the same speed and the gap between theoretical machine speed and actual utilization continues to increase.

Compilers are responsible for translating the abstract operational semantics of the source program, i.e., a text description of what the program's execution is supposed to perform, into an executable form that makes efficient use of a highly complex heterogeneous machine. Multiple architectural phenomena occur and interact simultaneously within the targeted computer during the execution of the program; this requires the optimizing compiler to combine multiple program transformations in order to define a program execution that takes advantage of those architectural phenomena. For instance, when targeting computers that have multiple processing elements (multi-core computers), there is often a trade-off between exploiting more processing elements simultaneously (parallelism) and exploiting data access locality to reduce memory traffic. Indeed, the speed and bandwidth of the memory subsystems are almost always a bottleneck. The problem is typically worse for multi-core computers. Since, in traditional compilers, optimization problems are associated with huge and unstructured search spaces, this combinational task is poorly achieved in general, resulting in poor scalability and disappointing sustained performance of the supposedly optimized program.

Generating efficient code for deep parallelism and deep memory hierarchies with complex and dynamic hardware components is a difficult task: the compiler (and run-time system) has to take the burden of tasks that only expert programmers would be able to carry. In order to exploit parallelism the first necessary step is to compute a representation which models the producer/consumer relationships of a program as closely as possible. The power of an automatic optimizer or parallelizer greatly depends on its capacity to decide whether two portions of the program execution may be interchanged or run in parallel. Such knowledge is related to the task of dependence analysis which aims at precisely disambiguating memory references. The issue is to statically form a compact description of the dynamic properties of a program. Forming a precise description is generally undecidable and approximations have to be made.

Once dependence analysis has been computed, a compiler performs program transformations to the code with respect to different, sometimes conflicting, performance criteria. Any program transformation must ultimately respect the dependence relations in order to guarantee the correct execution of the program. A class of transformations targeting the loop nests of a program (such as "DO" loops in the FORTRAN language, and "for" and "while" loops in languages derived from the C language) are known to account for the most compute intensive parts of many programs. The polyhedral model is a representation of a program's structure particularly suited for expressing complex sequences of loop nests, complex sequences of loop nest transformations, and other relevant information such as for instance dependences, communications, and array layouts.

A polyhedron is defined as a set of points verifying a set of affine inequalities and equalities on a number of variables. There exist alternate but equivalent definitions for polyhedrons, such as the one based on a combination of vertices, rays and lines proposed by Minkowski. There are also alternate representations, often based on the alternate definitions. While the present disclosure teaches using one of those definitions and representations to illustrate the various embodiments, various embodiments are in no way restricted to a particular definition or representation.

A polyhedral domain is defined as a finite union of polyhedrons. One of the main interests in using polyhedral domains is that they provide a precise representation of sets and relations among sets, on which many optimization problems can be phrased and solved using a rich set of algorithms, which are mostly available in the literature. Some embodiments of the sets in question represent loop iterations, mono- and multi-dimensional data sets, sets of processing elements, data transfers, synchronizations, and dependences. Thus, essential characteristics of the execution of a program can be summarized into compact mathematical objects, polyhedrons, which can be manipulated and transcribed into an executable program that has desired execution properties.

By considering a subset of the variables of a polyhedron as symbolic constants, also called "parameters", it is possible to perform program optimizations and parallelization as a function of the symbolic constants. Hence, programs involving loops that depend on a constant value that is not known at the time when compilation is performed, but only when the program is executed, can be modeled using polyhedrons that are defined as a function of those constant values. A polyhedron involving parameters is called a parametric polyhedron. Similarly, a parametric polyhedral domain is defined by a finite union of parametric polyhedrons. For instance, the set of values that the counters of a loop nest reach during the execution of the loop nest is represented by the loop nest's "iteration domain". The iteration domain of the following loop nest (using the C language's syntax, where F is a C function call) can be written as the parametric domain P(n): $\{(i, j) \in Z^2 | 5 \leq i \leq n; 0 \leq j \leq 10; j \leq i\}$:

```
for (i=5; i<=n; i++) {
    for (j=0; j<=i && j<=10; j++) {
        F(i,j);
    }
}
```

The set of iterations of such a loop nest depends directly upon the value of the parameters. The parametric domain that represents the set of iterations is called a "parametric iteration domain". It has to be noted that the values of the loop counters are integer. Hence, the set of values of i and j also lie on a regular lattice of integer points (the standard lattice Z^2 in the current example). However, it is possible to represent the fact that a set belongs to a polyhedral domain as well as the fact that it also belongs to a regular lattice of points using polyhedral domains exclusively. While alternate, equivalent representations exist (for instance those based on "Z-polyhedrons", which are an explicit intersection of a polyhedral domain and a lattice of integer points), various embodiments of the present invention are in no way restricted to exclusively using polyhedral domains. The use parametric polyhedral domains as a means to illustrate various provided embodiments. In some embodiments, either or both polyhedrons and Z-polyhedrons can be used as a representation, and there exist conversion methods between both representations.

While most of the transformations applied to the polyhedral representation of a program are defined for any element of the polyhedral domain to transform, a class of more complex and precise transformations is obtained by partitioning the vector space in which the polyhedral domain is defined into sub-polyhedrons, and by defining a different transformation for each polyhedron of the partition. The resulting transformation is called a "piecewise" transformation. For example, consider the transformation that takes two numbers i and j and computes three numbers x, y, and z as: {x=2i+1; y=(i+j)/2; z=−3j+4} when i is greater than j and {x=i; y=i−j+3; z=2j} when i is less than or equal to j. It is a piecewise affine function since it has different definitions for each set of values, {i>j} and {i<j}, which define a partition of the (i, j) vector space.

The context of various embodiments, the use of polyhedral representations to perform complex optimizations on programs, either independently or within a system of optimizing components. An exemplary embodiment of such a system is illustrated in FIG. **1**, where it is described as being part of a compiler. Flow of the exemplary embodiment starts in block **1**, where the compiler is processing a program. Flow continues in block **14**, where the compiler analyzes the program to decide if there are portions of the program that should be optimized and mapped using a polyhedral system. If it is the case, the flow goes to block **2**, where the compiler provides the system of optimizing components with a polyhedral representation of a portion of the program to be optimized. If not, the compiler continues to process the program without using the system of optimizing components and completes. The components of the system are in charge of a part of the global optimization of the input program. In the flow of the embodiment illustrated in FIG. **1**, the polyhedral representation of the input code is analyzed in block **2** to produce dependence information. Flow continues in block **3** where such information is used in a local memory compaction component or module that modifies array layouts in a way that removes some dependencies, schedules loop iterations in a way that exposes loops that scan independent iterations, and schedules the execution of operations using a same data to be executed within a close time interval. Flow continues in block **4**, where the modified polyhedral representation of the program is processed by another optimizing component, which partitions the represented loop operations into entities called tasks, which have good data locality properties (they access a data set that involves an optimized use of the memory subsystem of the target computer), and assigns a processing element of the target machine to each task. In this exemplary embodiment, the flow continues to decision block **5**, which decides which block is next in the flow as a function of the target machine. If the target machine requires the execution of explicit communication commands to transfer data to and from its processing elements, flow goes to block **6**, where the representation of the program thus modified is then processed by a series of optimizing modules which define a new layout for data that is transferred to a processing element's local memory. Otherwise, the flow goes to block **9**. From block **6**, flow continues to block **7**, where a representation of the explicit communications is produced, based on polyhedrons, and then to block **8**, where the execution of the communications are scheduled for parallel execution with the tasks, using multi-buffering. Whether the target machine requires explicit communications or not, the flow continues to block **9**, where an optimizing component processes the polyhedral representation of the program obtained from the previous components by inserting a polyhedral representation of synchronization operations, which ensure that the execution of the modified program produces the same results or similar results as the original input program. The flow of the exemplary embodiment then goes to block **11**, where an optimizing component partitions the tasks into subtasks whose execution reduces traffic between the processing elements' memories and their registers. Then, in block **12**, a polyhedral representation of commands that trigger the execution of a series of tasks on the different processing elements of the target machine and that wait for the completion of those, is generated by the next optimizing component. Finally, in block **13**, the polyhedral representation of the optimized program is transformed by polyhedral code generation component into a representation (Abstract Syntax Tree, high-level language code, or a compiler's internal representation) that can be either processed by a compiler or processed further by the user. In the exemplary embodiment, the flow continues back to block **1**, where it may cycle again through the whole flow if there is more code to be optimized using the system of optimizing components.

In contrast to compilers based on polyhedral domains, traditional loop-oriented optimizing compilers typically perform syntactic transformations. As a result, many interesting optimizations are often not available, such as fusion of loops with different bounds or imperfectly nested loop tiling.

In some embodiments, the optimizing components or modules comprise processor executable code that when executed by a processor, convert source code into other forms of source code, or in some instances machine code. In other embodiments, various modules may be implemented in hardware such as monolithic circuits, Application Specific Integrated Circuits (ASIC), or Field Programmable Gate Arrays (FPGA). These modules may comprise software, hardware, firmware, or a combination of these implementations. It is important to note that various embodiments are illustrated in specific programming languages, these illustrations are mere examples and the scope is not therefore limited to any particular programming language.

Embodiments of a provided optimization module, described above as local memory compaction are illustrated in FIGS. **2-8**. FIG. **2** illustrates the flow of a provided method for local memory compaction. Flow begins in block **10** where source code is received into memory. In this embodiment, the source code represents loops with arbitrary parametric affine iteration domain and contains at least one array reference. An array reference is an operation that represents an access, typically a read or a write, to an array. Such a reference represents, either explicitly or implicitly, for instance by using programming language conventions, a function to retrieve the memory address of an element of the array. In loop programs, that function is typically a direct or indirect function of the loop indices and of some loop-constant values. For instance, in C, arrays are typically referenced through mono- and multi-dimensional affine functions of some input values. In the C language, the declaration of an array includes parameters called "array size", which implicitly define the address of an array element as a function of the input values to references to this array, declaring "char A[100][200]" allocates an array of 20000 elements (100×200), named A, and defines that for any two integer values x and y, the memory address of the element of A referenced through A[x][y] is b+200x+y, where b is a value called the "base address" of array A. b is constant for each array and is determined at some point in the compilation process. Flow continues to block **20** where inefficiencies in memory usage in the at least one array are identified. In one embodiment, the inefficiencies are related

to access and memory footprint of the array. Flow then continues to block **30** where at least one local array is allocated, and in block **40** a portion of the array with inefficient memory usage is mapped into the local array. The mapping portion of the module outputs code that is more efficient than the original code in terms of the memory size requirements of the local array versus the original array. In some embodiments the accessed data is arbitrarily complex. In further embodiments, the mapping produces a piecewise affine index function for the local arrays. Other embodiments include the rendering of a visualization of the optimized code on a monitor.

Arrays are typically allocated sets of contiguous memory blocks. Some loop operations may access only portions of the allocated memory. When reorganizing the data layout for a specific processor, there is an opportunity to take advantage of the inefficiencies in memory access requirements versus the actual utilization of the array. For example, given the following code fragment, 900,000 contiguous memory blocks are allocated, but only 100 are accessed in this operation. Furthermore, access to the array is not contiguous, but contains gaps, and thus will have less than optimal locality. Thus keeping the original data layout (and array size) in a remote processor is extremely inefficient. Moreover, if there are less than 900,000 blocks available in the local memory, the local memory cannot hold the entirety of the array and the program cannot be executed properly. In the provided code fragments, we are using to elude other operations which do not have any specific illustrative purpose.

```
double A[300][300];
for (i=0; i<100; i++) {
. . . = . . . A[2*i+100][3*i]; }
```

One embodiment of a provided method, illustrated in FIG. **2**, would map this code fragment into a local array with 100 elements. An exemplary mapping would produce the following pseudo-code fragment, in which the storage requirement of a local array is reduced from 300×300 elements to the optimal 100 elements.

```
double A_local[100]; //local memory
transfer A[2*i+100][3*i] to A_local[i], i=0, 1, . . . 99;
for (i=0; i<100; i++) {
. . . = . . . A_local[i]; }
```

One feature of this embodiment is that it provides a method of compacting local memory in a computing apparatus. This method provides a more efficient memory structure in terms of both access to the elements and the amount of memory occupied by the data that is actually accessed. The memory requirements are reduced from the initial allocation to an allocation that is large enough to contain the data that is actually used in the operations. In contrast to other methods, the provided method handles loops whose iteration domains are non-rectangular, and loops that have a parametric iteration domain. In this document we refer to polyhedral iteration domains that are either non-rectangular or parametric or both as "arbitrary parametric iteration domains". In addition, the provided methods handle non-convex accessed data sets. The provided embodiments are very useful in image and video processing. Imaging applications typically utilize significant multi-dimensional arrays where data representations of physical objects and systems are stored. Many image processing steps, such as discrete wavelet transforms for example, only utilize discrete portions of the stored data. In these situations, various embodiments provide significant optimizations to local data storage.

Another embodiment of a provided method is illustrated in FIG. **3**. In this embodiment, flow begins in block **10** where source code is received in memory. Similar to the above embodiment, the source code contains loops with arbitrary parametric iteration domain and contains at least one array reference. Flow continues to block **20** where inefficiencies in memory usage in the at least one array are identified. Flow then continues to block **30** where at least one local array is allocated, and in block **40** a portion of the array with inefficient memory usage is mapped into the local array. In this embodiment, mapping block **40** includes partitioning references to form compatible references in block **50**; determining a relation within compatible references in block **60**; grouping compatible references based on the relation in block **70**; performing algebraic simplification in block **80**; and performing geometric arrangement through re-indexing the elements of the local array in block **90**. In some embodiments the set of references partitioned are references that access a portion of the array. The following pseudo-code example illustrates this embodiment.

```
float A[256] [256] ;
doall (l=128*j+16*P; l <= min(−i+254,128*j+16*P+15); l++)
    doall (m = 16*k; m <= min(−i+254, 16*k+15); m++)
        A[1+i+m] [1+i+l] −= A[1−i+m] [i] * A[i] [1+i+l];
```

In this case, all three references to array A are disjoint in that they access disjoint portions of the array. In this case, they are transformed into three local arrays A_2, A_3 and A_4 in the following manner.

```
float A_2[16] [16]; // a triangular subregion of A
float A_3 [ 16]; // a column of A
float A3 [16]; //a row of A
doall (l = 0; l <=min(15, −i−128*j−16*P+254); l++)
    doall (m = 0; m <= min(−i−16*k+254, 15); m++)
        A_2[m] [I] −= A_3[m] * A_4[l];
```

Performing transformations of the way data are allocated in memory, i.e., transforming the data layouts, has a combinational aspect, since the data sets accessed through each array reference may overlap with one or more data sets accessed by other array references. Since each one of those overlaps entail constraints in the way that data layouts can be transformed, analyzing all the combinations of overlaps for all the references is a source of high computational complexity. Hence, references are grouped into sets in such a way that data accessed through one set of references does not overlap data accessed through another set of references. In this embodiment, references of the same set are called "compatible references". Since there is no overlap among sets of compatible references, the following parts of the memory layout transformation, which consider the overlaps, can be applied independently to each set of compatible references. In particular, they will decide if the overlapping data sets accessed by a set of compatible references should be partitioned further and how.

In some embodiments, compatible references are identified by overlapping memory footprints during the execution of a particular subset of loop iterations. In an exemplary embodiment, the provided method identifies array references having overlapping memory footprints; duplicates a

portion of the identified references; and associates each of the duplicates with disjoint subsets of the memory footprint. An example pseudo-code illustrates this embodiment.

```
double A[ 100] [100];
for (j = 0; j<100; j++) {
    . . . = A[i] [j] * A[j] [i];
}
```

The two references A[i] [j] and A[j] [i] overlap when H. However, if the references are allocated together, it is impossible to reduce the local memory usage using only affine transformations. This is because the data footprint of the two references is a 2-dimensional set (a cross), while the data footprints of the individual references are both 1-dimensional. In order to compute better allocations in situations like this, one embodiment first estimates how much overlapping is in the references. If the references are read-only, and if the overlapping data set is a small percentage of the overall data set, the embodiment splits the references into two distinct references to one-dimensional data sets. In the above example, the embodiment will generate the following local memory allocation. Note that the center element of the data foot print, A[i][i], has been replicated and put into the locations A_1 [i] and A_2 [i].

```
double A_1[100];
double A_2[100];
Transfer A[i] |j] to A_1[i], i = 0 . . . 99
Transfer A[j] [i] to A_2[i], i = 0 . . . 99
For (j 0; j < 100; j++)
    . . . A_1[j] * A_2[j];
```

The geometric re-arrangements provided by a further exemplary embodiment are defined by a piecewise affine transformation. In other words, the transformation applied to the references is defined as a set of functions, each element of the set being valid within a polyhedral domain of the loop values, the parameters and the coordinates of the data accessed through the set of compatible references. In an exemplary embodiment, when some of the data accessed by a set of compatible references are written by some of the references, the written data subset and a subset of the data set that is only read define a partition for the piecewise affine transformation. Consider the program represented by the following pseudo-code:

```
double A[100][100];
for (j = 0; j < 99; j++) {
        A[i] [j+1] =. . . A[j] [i];
}
```

In this example, the data set accessed by the both references to array A form a two-dimensional set, while the data sets accessed through each reference are one-dimensional. The data accessed through both references overlap in A[i][i]. In the exemplary embodiment, a piecewise transformation of A is applied, which separates A into two subsets, one for each one-dimensional data set, and marks one of them as receiving the updates (let us call it the "writing reference") to the duplicated data. In the example, the duplicated data is A[i][i] and the iteration domain is partitioned into three polyhedral domains, {0≤j<i}, {j=i} and {i<j<99}, in order to take into account the fact that only one of the data subsets is updated. Such a partition of the iteration domain is obtained by defining the iterations accessing duplicate data through "non-writing" references and replacing those accesses with an access through the writing reference. The resulting piecewise affine transformation is {(A[i][j−1]=A_1[j], A[j][i]=A_2[j]) for 0≤i<100, 0j≤i or i<j<100; and (A[i][j−1]=A_1[j], A[j][i]=A_1[j]) for 0≤i<100, i=j}. The result of the piecewise affine transformation can be represented by the following pseudo-code, which uses only two arrays as a replacement for the original array A, has quasi-optimal memory requirements (198 memory cells, while the optimal would be 197):

```
double A_1[99], A_2[99]
for (int j=0; j<i; j++) {
    A_1[j] = . . . A_2[j];
}
A_1 [i] = . . . A_1 [i−1 ]; // the updated value of A[j][i] is in A_1 [j]
when j=i
for (int j=i+1; j< 99; j++) {
    A_1[j] = . . . A_2[j];
}
```

In other exemplary embodiments, the geometric rearrangement is a piecewise affine transformation that defines a partition of the iteration domain and of the data sets in such a way that the number of references to a local array varies from one element of the partition to another. In the following example, in which the possible values of variable i are {0<i≤99900}, the data sets accessed through reference A[j] and A[i+j] overlap when i is less than 100. Otherwise, they do not overlap.

```
double A[10000];
for (j =0; j< 100; j++) {
    A[i] = . . . * A[i+j]
}
```

Since those data sets overlap for some values of i, both references are put in the same group of compatible references. If the accessed data sets are allocated as a single local array, the amount of memory necessary to contain the array is 10000 memory cells. On the other hand, if they are allocated as two separate arrays, some of the data would have to be duplicated and the iteration domain (the j loop here) would have to be partitioned as in the previous exemplary embodiment. The amount of overlap when i is less than 100 may not be small enough and it may not be profitable to perform the duplication. The geometric rearrangement provided by the embodiment is a piecewise affine transformation that defines a partition of the set of parameters (in the current example, i): {(A_1[j]=ADD for 0≤i<100, and (A_1[j]=A[j], A_2[j]=A[i+j]) for i≥100}. The maximum amount of memory that has to be allocated for any value of i is 200 memory cells (as compared to 10000), and it is 100+i when i is less than 100. The resulting transformation can be represented as pseudo-code as follows:

```
if (i <100) {
    double A_1 [100+i];
    for (j=0; j< 100; j++) {
        A_1[j] = . . . * A_1[i+j]
}
else {
    double A_1 [100];
    double A_2[100];
    for (j=0; j<100; j++) {
    A_1[j] = . . .*A_2[j];
}
}
```

One advantage of the geometric rearrangement that is performed by this exemplary embodiment is that the j loops are not partitioned. Partitioning the loops into smaller loops is often a factor of performance degradation, which is avoided in this exemplary embodiment. The partition of i is obtained by computing the domain in which both data sets intersect, by projecting the intersection onto the vector space of the parameters (in the current example, the parameter is i and the projected domain is {i<100}.

The operation flow of a further provided embodiment of a local memory compaction module is illustrated in FIG. **4**. In this embodiment, flow begins at block **10** where source code is received in memory. Similar to the above embodiment, the source code represents loops with arbitrary parametric affine iteration domains and contain at least one array reference. Flow continues to block **20** where inefficiencies in memory usage in the at least one array are identified. In this embodiment, the identification of inefficiencies includes block **100** where strides in the polyhedral domain that defines the accessed dataset are identified, and block **110** where a lattice of integer points within the domain is extracted from the domain. These integer points indicate that only a regular subset of the accessed data region is accessed. In this manner, more efficient allocation of local arrays is accomplished because portions of the array that are not accessed are identified and excluded from the mapping from the array to the local array.

An additional provided embodiment is illustrated in FIG. **5**. In this embodiment, like earlier embodiments flow begins at block **10** where source code is received in memory. Similar to the above embodiment, the source code represents loops with arbitrary parametric affine iteration domain and contains at least one array reference. Flow continues to block **20** where inefficiencies in memory usage in the at least one array are identified. Flow then continues to block **30** where at least one local array is allocated, and in block **40** a portion of the array with inefficient memory usage is mapped into the local array. In this embodiment, like in the embodiment illustrated by FIG. **3**, mapping block **40** includes partitioning references to form compatible references in block **50**; determining a relation within compatible references in block **60**; grouping compatible references based on the relation in block **70**; performing algebraic simplification in block **80**; and performing geometric arrangement in block **90**. The algebraic simplification block **80** includes block **130** where a representative array reference is extracted from a set of references accessing a portion of the array. In some embodiments, the representative array reference represents a set of references which access polyhedral datasets whose accessed points all lie on a lattice of integer points that is not the standard lattice, on which any integer point lies. These embodiments take advantage of the fact that array references represent affine functions, which can be represented as matrices called "access matrices". In the exemplary embodiment, the flow within block **40** goes from block **130** to block **140** where a Hermite factorization is performed for the access matrix representing the representative array reference. The Hermite factorization produces a piecewise affine index function.

One purpose of Hermite factorization is to reduce the dimension of the reference to the actual geometric dimension of the data footprint. In addition, if the access pattern contains strides, i.e., regular intervals between accessed data, using the non-unimodular matrix that results from the Hermite factorization in the transformation removes these strides in the resulting local references. For example, given an affine access function f(x, y) on loop indices x and parameters y, we first decompose it into the sum of g(x)+h(y), where g(x) is a linear function on x and h(y) is an affine function on y. This decomposition is an algebraic simplification that makes it possible to perform further computations on the part of f(x,y) that involves variables only. Function g(x) can be decomposed into g(x)=HU, where H=[H' 0] is the Hermite Normal Form of g(x) and U is unimodular matrix. Let

$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

where $HU=H'U_1$. The following mapping from global to local indices is then performed $f(x, y)f \rightarrow U_1 x$.

Hermite factorizations have many uses is lattice computations. The Hermite factorization of a matrix G, written G=HU, writes matrix G as the product of two matrices, H and U. H, called the "Hermite normal form", is a canonical representation of the lattice (also) represented by G. U is a unimodular matrix, which entails that U, when used as a transformation, always transforms any point that has integer coordinates into another point that has integer coordinates. Also, any point that has integer coordinates can be obtained by transforming a point with integer coordinates using a unimodular transformation. This is important since most programming language conventions enforce that data elements, and particularly array elements, must have integer coordinates.

The flow of a still further provided embodiment is illustrated in FIG. **6**. In this embodiment, like previous embodiments, flow begins at block **10** where source code is received in memory. Similar to the above embodiment, the source code represents loops with arbitrary parametric affine iteration domain and contain at least one array reference. Flow continues to block **20** where inefficiencies in memory usage in the at least one array are identified. Flow then continues to block **30** where at least one local array is allocated, and in block **40** a portion of the array with inefficient memory usage is mapped into the local array. In this embodiment, mapping block **40** includes partitioning references to form compatible references in block **50**; determining a relation within compatible references in block **60**; grouping compatible references based on the relation in block **70**; performing algebraic simplification in block **80**; and performing geometric arrangement in block **90**. Geometric rearrangement **90** contains blocks **150** where linear constraints are formed, block **160** where sets of linear programming problems are formed from the linear constraints and solved, and block **170** where a data reindexing is computed. In some embodiments, the flow goes back to block **150**. In such embodiments, geometric rearrangements are applied iteratively until no reindexing function is found that reduces memory requirements.

Most modern programming languages abide by the convention that multidimensional arrays are allocated in memory as if they were canonical rectangular parallelotopes. In a space of d dimensions, a parallelotope is a finite polyhedron defined by 2d faces, and whose faces are pairwise parallel. A canonical rectangular parallelotope is a parallelotope for which the normal vectors to its faces are either a canonical vector or the negation of a canonical vector. Examples of rectangular parallelotopes are a cube (in a 3-dimensional space) and a rectangle (in a 2-dimensional space). In an exemplary embodiment, the transformation is a unimodular reindexing of the accessed data that minimizes

the size of the smallest canonical rectangular parallelotope that encloses the accessed dataset. The smaller the enclosing rectangular parallelotope, the smaller the amount of memory that has to be allocated for the dataset.

In some embodiments, this is accomplished by formulating a first set of linear constraints through the use of Farkas Lemma. This first set of linear programming constraints is decomposed dimension by dimension to form a set of integer linear programming problems. This set of problems is then solved to provide the data reindexing function which can then be applied to the at least one local array. Unimodular reindexings transform integer points into integer points. Hence, the convention that data elements have integer coordinates is preserved by such a reindexing. In the case of affine transformations, the linear part of the transformation can be represented by a unimodular matrix.

Farkas lemma is a basic linear algebra theorem which is often used to obtain, from a set of affine constraints (i.e., inequalities and equalities) on variables with unknown coefficients, constraints that apply to the unknown coefficient themselves. In this embodiment, it is used to obtain a set of constraints involving the coefficients of the unimodular data reindexing function (which is represented as a matrix) and the width of the enclosing rectangular parallelotope along each dimension. From those obtained constraints, the method embodiment finds values of the coefficients of the unimodular data reindexing function for which the width is minimal, using integer linear programming. For example, the data set accessed through reference B[i+j][j] in the following pseudo-code can be reindexed so as to occupy only 100 memory cells:

```
Double A[n+10][n+10];
Double B[[2n+20][n+10];
For (i=n; i<n+10; i++) {
    For (j=n; j<n+10; j++) {
        A[i]j] = . . .B[i+j][i];
    }
}
```

The coordinates (x_1,x_2) of the elements of array B accessed by that loop node are defined by the constraints $D:\{n\leq x_2<n+10;\ n\leq x_1\ n+10\}$. The embodiment finds values of the coefficient of a matrix U such that U is unimodular and the coordinates x'−1 and x'2 of the reindexed data are defined by:

$$\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = U\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} n + \begin{bmatrix} t_{01} \\ t_{01} \end{bmatrix}$$

The set of possible values of the coefficients of U, as well as the possible values of t1, t2, t_{01} and t_{02} are defined from the set of constraints D and the constraints that the data (x'_1,x'_2) are enclosed in a rectangular parallelotope of size (s_1, s_2) using Farkas lemma. Then, a value for those coefficients is computed for which the size of the smallest enclosing rectangular parallelotope (s_1, s_2 in our example) is minimal. Those values are computed by solving, dimension by dimension of the data set, an integer linear programming problem.

An integer linear programming problem defines a linear function of a set of variables, called the "objective function" and whose minimal (or, alternatively, maximal) value over a polyhedral domain called the "feasible set", is looked for. Solvers for such problems typically return a polyhedral domain, within the feasible set, for which the value of the objective function is minimal. In the running example, the embodiment finds:

$$\begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} n + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The following pseudo-code represents the program resulting from the data reindexing of array B in our running example:

```
Double A[10][10];
Double B[[2n+20][n+10];
For (i=n; i<n+10; i++) {
    For (j=n; j<n+10; j++) {
        A[i]j] = . . .B[j−n][i−n];
    }
}
```

The data footprint of the re-indexed array B is now reduced to 100 memory cells, instead of $n^2+20n+100$ initially.

In one of the exemplary embodiments, the unimodular nature of the reindexing matrix U is obtained by forcing U to be triangular and forcing the absolute value of the diagonal elements to be one. In another embodiment, the unimodular nature of the reindexing matrix is obtained by composition of an upper triangular unimodular and a lower triangular unimodular matrix. The advantage of that other embodiment is that the class of unimodular reindexing functions produced is not limited to the reindexing functions represented by a triangular matrix. Finding those two matrices is equivalent to reindexing data twice, first by finding an upper triangular reindexing matrix as described above and applying the reindexing, and then by finding a lower triangular reindexing matrix for the reindexed set and by applying that second reindexing. Yet another embodiment produces, in the same way, a unimodular reindexing by composition of an upper triangular unimodular matrix, a permutation matrix and a lower triangular unimodular matrix. The advantage of the embodiment is that the class of reindexing function that can be produced is the whole class of integer unimodular matrices.

Turning to FIG. **7** which illustrates another embodiment of a provided method, like the previous embodiments, flow begins in block **10** where source code is received in memory. Similar to the above embodiment, the source code represents loops with arbitrary parametric affine iteration domain and contains at least one array reference. Flow continues to block **20** where inefficiencies in memory usage in the at least one array are identified. Flow then continues to block **30** where at least one local array is allocated, and in block **40** a portion of the array with inefficient memory usage is mapped into the local array. In this illustration, block **40** contains block **180** where a parallelotope of minimal volume is derived this parallelotope enclosing the domain of the data set accessed by the local arrays. Block **40** additionally contains block **190** where a finite prism of triangular base is derived.

As used herein, a finite prism is a polyhedron defined by a set of translations of a "base" polyhedron, which lies in a subspace of the considered space, by a finite convex set of linear combinations of vectors of the complementary subspace. Since they are finite, it is possible to characterize the maximum extent of a finite prism along the directions of the complementary subspace. In this document, those extents are called "height" of the prism (there is one height along

every direction of the complementary subspace). A triangular prism is a prism whose base polyhedron is a triangle. In two dimensions, it is just a triangle. In one embodiment, this finite prism has a minimum volume that encloses the data footprint domain. In block **200** the prism is compared to the parallelotope. In block **210** the prism is partitioned into two prisms. One of the two is then transformed using a central symmetry such that the union of the transformed prism and the non-transformed prism has a smaller memory footprint than the enclosing parallelotope. One advantage of that embodiment is that it provides data layouts that have smaller memory requirements, for a class of accessed datasets for which methods based on parallelotopes are not optimal.

For instance, the dataset accessed by the program represented by the following pseudo-code through reference B is triangular:

```
For (i=0; i< 10; i++) {
    For (j=0; j< i; j++) {
        . . . = . . . B[i][j];
    }
}
```

The embodiment finds three constraints that enclose the accessed data set, in a similar way as in the embodiment depicted in FIG. **6**, using the Farkas lemma. The minimal volume for a parallelotope that encloses the dataset would be about twice the volume of the triangle. Hence, using such a parallelotope to determine the memory allocation of the dataset is bound to be sub-optimal. Instead, the current embodiment, depicted in FIG. **7**, defines a tighter enclosing polyhedron using three inequalities (it is then a prism of triangular base). Using the enclosing prism, the data set is partitioned in two subsets, say A and B, and subset A is re-indexed in such a way that both the array elements in B and the re-indexed elements are enclosed in a smaller parallelotope than the original parallelotope. The volume of the new parallelotope is about the volume of the prism of triangular base. Since there is a parallelotope of smaller volume enclosing the reindexed data set, its memory requirements are smaller. The result is a piecewise affine array reindexing, which typically partitions the loop iterations into the iterations that access A, and the ones that access B.

In the current embodiment, the three inequalities {(a): $al+a_0 \geq 0$; (b):$bl+b_0 \geq 0$; (c): $cl+c0 \geq 0$} that define the triangular prism P, where I is the vector of data coordinates are used to devise the partitioning. Let x_w a point in the intersection of (b) and (c) and let $w=ax_wI+a_0$. The prism is partitioned into A and B as follows:

$$A = P \cap \left\{ aI + a0 - \frac{w+1}{2} \geq 0 \right\}$$

and B=P−A. A point, x_0, is defined that is in the domain $\{al+a_0-w+1<0;\ bl+b_0<0\}$ whose coordinates are a multiple of ½ and whose "height" in the prism is about half of the height of the prism. Array elements that are defined by A are transformed using a central symmetry of center x_0. In the program represented by the following pseudo-code, the tightest enclosing parallelotope, defined by $\{0 \leq x1 \leq 9;\ 0 \leq x2 \leq 9\}$, where x**1** represents the first dimension of array C and x2 its second dimension, includes 100 array elements.

```
Double C[10][10];
For (i=0; i< 10; i++) {
    For j=0; j< i; j++) {
    . . .C[i][j]. . .;
    }
}
```

The tightest enclosing triangle, defined by $\{0 \leq x1;\ 0 \leq x2;\ x1+x2 \leq 9\}$, by comparison, includes 55 elements, which is about half the number of elements required for the enclosing parallelotope. Since the number of array elements in the enclosing triangle is less than the number of array elements in the enclosing parallelotope, the embodiment considers the tightest enclosing triangle and partitions the enclosed data into data subsets A: $\{0 \leq x_1;\ 5 \leq x_2;\ x_1+x'_2 \leq 9\}$ and B: $\{0 \leq x_1;\ 0 \leq x_2 \leq 4;\ x_1+x_2 \leq 9\}$. Point $x_0=(5, 9/2)$ is selected as center of symmetry and the elements of A are then transformed into a new array subset A' as follows: $\{x'_1=(2*5)-x_1;\ x'_2=(2*9/2)-x_2\}$, where (x'_1,x'_2) are the new array element coordinates. The resulting program can be represented by the following code:

```
Double C[11][5];
For (i=0; i< 10; i++) {
    For (j=0; j<=4 &&j<i; j++) {
        . . .C[i][j]. . .;
    }
    For (j=5; j<i; j++) {
        . . .C[10-i][9-*j]. . .;
    }
}
```

The accessed data set is included in the parallelotope $\{0 \leq x1 < 11,\ 0 \leq x2 < 5\}$, whose memory requirements are of 55 memory cells, i.e., about half of the parallelotope before the transformation. Other data layout optimizations, which are optimal when there is a tight parallelotope enclosing the accessed dataset, will then be more optimal than if applied straightforwardly to the original dataset that can be enclosed more tightly with a triangular prism.

FIG. **8** illustrates a further embodiment of a provided method. In this embodiment, flow begins in block **10** where source code is received in memory. Similar to the above embodiment, the source code contains loops with arbitrary parametric affine iteration domain and contain at least one array reference. Flow continues to block **20** where inefficiencies in memory usage in the at least one array are identified. Flow then continues to block **30** where at least one local array is allocated, and in block **40** a portion of the array with inefficient memory usage is mapped into the local array. Flow then continues to block **220** where asynchronous communications and wait operations are generated. The exemplary embodiment uses the mapping between local memory elements and the elements of the original arrays, in conjunction with a description of the data that are needed as input and produced as output of the tasks to be executed, to produce an abstract representation of the transfers between the original arrays and the local memory elements. In an exemplary embodiment, the generation of these communications and wait operations includes the use of multi-buffering for overlapping communication and computation operations.

Many computers that contain processors that have an explicitly managed local memory also have the ability to transfer data at the same time as they are performing other computations. Such transfers are called "asynchronous". The main reason for using that feature is that the typical time

necessary for such transfers is often comparable to the time taken to perform computations between two consecutive transfers of input data. Since doing both transfer and computation at the same time takes less time than doing one after another, the effect of overlapping them is to improve the overall program execution time. The use of several memory zones, specialized to either execution, reception or sending of data, makes the overlap possible. Such a use is called "multi-buffering". The specialization of the buffers is also modified at certain times. Such a modification is called a "rotation of the buffers", since a buffer is cyclically assigned the same specialization.

One embodiment computes a local memory mapping, adds a polyhedral representation of the communications and schedules communications and computations in a multi-buffering scheme for the program represented by the following pseudo-code. In this pseudo-code, every iteration of the k loop works on a distinct instance of local memory:

```
        for (k = 0; k <= 7; k++) {
            for (l = 0; l<= 15; l++) {
                for (m = 0; m <= 15; m++) {
                    for (n = 16 * k; n <= 16 * k + 15; n++) {
                        C[l][m] = C[l][m] + A[l][n] * B[n][m];
                    }
                }
    }
}
```

This results in a program that can be represented by the following pseudo-code:

```
for(k = -1; k<= 8; k++) {
        if (k <= 7 && k >= 0) {
            wait(tag=0);
              rotate(vars=[C_l, A_l, B_l]);
        }
        if (k <= 6) {
            for (l = 0; l <= 15; l++) {
                for (m = 0; m <= 15; m++) {
                    get(src=&B[l][16 + 16 * k + m],
                    dst=&B_l<1>[l][m], tag=0);
    }
}
for (l = 0; l <= 15; l++) {
    for (m = 0; m <= 15; m++) {
        get(source=&A[l][m], destination=&A_l<1>[l][m], tag=0);
    }
}
for (l = 0; l <= 15; l++){
    for (m = 0; m <= 15; m++) {
        get(src=&C[l][16 + 16 * k + m, tag=0);
    }
  }
}
if (k >= 1) wait(tag=1);
if (k <= 7 && k >= 0) {
    for (l = 0; l <= 15; l++) {
        for (m = 16 * k; m <= 16 * k + 15; m++) {
            for (n = 0; n <= 15; n++) {
                C_l[l][-16 * k + m] = C_l[l][-16 * k +m] +
                B_l[n][-16 * k + m] * A_l[l][n];
            }
        }
    }
    for (l = 0; l <=15; !++) {
        for (m = 0; m <= 15; m++) {
            put(src=&C_l[l][m], dst=&C[l][16 * k + m], tag=1);
        }
        }
    }
    }
```

In the code example, "Get" operations are transfers from an original array to a reindexed array in local memory. "Put" operations are transfers from local memory to original array. While the values of k in the original program were going from 0 to 7, in the multi-buffered version produced by the embodiment they are going from −1 to 8. At iteration k=−1, the first "get" transfers are issued. At iteration k=8, the last "put" transfers are issued. "Wait" operations, which wait for the completion of a series of transfers, were also inserted to ensure that a transferred data set is completed at that point of the program's execution. In the embodiment, a tag system is used to identify the transfers whose completion is to be waited upon. The "rotate" operations operate the buffer rotation.

Automatic Generation of Data Transfer

An additional feature of various embodiments is that they provide for automatic generation of communication operations (e.g., asynchronous DMA) to transfer data between two memories (e.g., local and global memories). Some provided embodiments can convert communication commands, which have the semantics of an element-per-element copy, into strided DMA commands, which can copy (strided) blocks of data in an efficient manner and thus actually exploit the bulk communication capabilities/requirements of the target machine. The algorithm typically works by aggregating one-dimensional, polyhedral sets of data into packets, and produces a loop that scans these packets. The algorithm then generates a data transfer operation, e.g., a DMA command (with its parameters) that will execute the packet transfer. These operation parameters (e.g., DMA command parameters) are usually the base address, the stride, the element size and the number of elements to communicate, for both the sender and the receiver sides. Further embodiments provide methods for generalization to different data transfer capabilities and restrictions by a number of methods including two sided (e.g., remote and host) strides, strides in more than one dimension, and accounting for alignment constraints and memory bank constraints. In some instances, it is desirable to enlarge the transfer set so that it becomes rectangular, rather than of a complex, of a non-convex shape. This is similar but not the same as what is provided in local memory compaction's unimodular reindexing described above. Further embodiments provide for merging multiple data transfer commands into a single command. For example, if two transfers happen to be from and to contiguous memory, or if one command transfers data from even addresses and another to odd addresses, the corresponding data transfer operations may be combined into a single data transfer operation.

In general, tasks that are mapped onto different processors communicate with each other and with a main host processor. When migrating data from one processor to another, we have a further opportunity to change the data layout to improve storage utilization and locality of references. For example, when inserting explicit communication into the following loop:

```
double A[300,300];
    for (i = 0; i < 100; i++) {
    A[2*i+100,3*i] ... ;
}
```

we can utilize a different layout the array A in local memory as shown below:

```
double A_local[100];
Transfer data from A[2*i+100,3*i] to A_local[i]
for (i = 0; i < 100; i++) {
    . . . = . . . A_local [i] . . .;
}
```

As described above, transforming the reference from A [2i+100, 3*i] to A_local[i] can reduce the storage requirements of local memory from 300×300 elements to 100 elements.

Thereafter, communication commands created in the mapping process may have to be physically realized as actual operations supported by the underlying architecture. In some embodiments, this is performed during data transfer operation generation/optimization phase of the mapper. On some architectures such as CELL and TRIPS, communication commands are mapped into operations to the DMA engines issued from the remote processors. On other architectures, such as some FPGA systems, they are turned into buffer management and DMA operations to be issued on a host processor.

Generating Bulk Communication

In various embodiments, one of the tasks of the local memory compaction phase is to insert communication code to transfer data between two memories, such as global and local memories. In order to ensure communication overhead is minimized, only bulk communication operations are generated, and these are inserted only at the boundaries of the innermost kernels, such that there is no communication operations within a kernel. During communication insertion an asynchronous communication model is assumed with the following primitives:

Asynchronous communication initialization operations such as put and get. These operations can initiate an asynchronous transfer of a region of data from one processor and/or memory to another. These generic operations can be expressed as:

get A[f(x,y)] from B[g(x,y)] for x$\in\mathcal{D}$(y) tag t

put A[g(x,y)] to B[f(x,y)] for x$\in\mathcal{D}$(y) tag t

where A is a local array, B is a global array, y are the system parameters, and (f,g,$\mathcal{D}$) together describe the set of elements to be transferred. The semantics of the get operation is identical to the following loop nests:

for x$\in\mathcal{D}$(y)

A[f(x,y)]=B[g(x,y)]

while the put is identical to the following:

for x$\in\mathcal{D}$(y)

B[f(x,y)]=A[g(x,y)]

A wait t operation which blocks the execution of a thread until a group of initiated communications have completed. Groups of communication operations are associated with a tag t.

It should be stressed that the above primitives are generic in nature, and do not necessarily correspond directly to actual target machine primitives. Mapping of these primitives into lower level DMA operations is described later. The following example illustrates how communication operations can be inserted by a mapper in one embodiment. Suppose the following matrix multiply is to be mapped onto a distributed memory machine that includes eight (8) distributed processor elements:

```
float A[1024][1024];
float B[1024][1024];
float C[1024][1024];
for (int i = 0; i <= 1023; i++) {
    for (int j = 0; j <= 1023; j++) {
        for (int k = 0; k <= 1023; k++) {
            C[i][j] = C [i][j] + A [i][k] * B [k][j];
        }
    }
}
```

After affine scheduling, tiling and processor placement, we can obtain the following single-program, multiple data (SPMD) loop nests, where the newly introduced parameter P ranges from 0 to 7 and stands for the current processor id.

```
float A[1024][1024];
float B[1024][1024];
float C[1024][1024);
for (int i = 0; i <= 31; i++) {
    for (int j = 128 * P; j <= 128 * P + 127; j++)
        for (int k=32 * i; k<=32 * i + 31; k++)
            c [j][k] = 0;
    for (int j = 0; j <= 15; j++)
        for (int k = 128 * P; k <= 128 * P + 127; k++)
            for (int l = 32 * i; l <= 32 * i + 31; l++)
                for (int m = 64 * j; m <= 64 * j + 63; m++)
                    C[k][l] = C [k][l] + A [k][m] * B [m][l];
}
```

The above loop nests operate on arrays A, B, and C, which are located in the memory of the host processor. In order to produce a distributed memory mapping, local memory compaction optimization is performed as described above and the following loop nests are produced. New local variables A_I, B_I and C_I are introduced and placed within the memories of the remote processor units:

```
local float A_l[128][64];
local float B_l[64][32];
local float C_l[128][32];
float A[1024][1024];
float B [1024][1024];
float C [1024][1024);
for (int i = 0; i <= 31; i++) {                          // LOOP i
    for (int j = 128*P; j <= 128*P + 127; j++)
        for (int k = 32*i; k <= 32*i + 31; k++)
            C_l[ j -128*P] [-32*i + k] = 0;
    put C_l[j][k] to C[j + 128*P][32*i + k],
        for 0 <= j <= 127, 0 <= k <= 31 tag 1;
    wait 1;
    for (int j = 0; j <= 15; j++) {
        // fetch submatrices from A, B and C
        get B_l[k][l] from B[64*j + k][32*i + l]          // GET B
            for 0 <= k <= 63, 0 <= l <= 31 tag 0;
        get A_l[k][l] from A[k + 128*P][64*j + l]         // GET A
            for 0 <= k <= 127, 0 <= l <= 63 tag 0;
        get C_l[k][l] from C[k + 128*P] [32*i + l]        // GET C
            for 0 <= k <= 127, 0 <= l <= 31 tag 0;
        wait 0;
        // kernel
        for (int k = 128*P; k <= 128*P + 127; k++)
            for (int l = 32*i; l <= 32*i + 31; l++)
                for (int m = 64* j; m <= 64*j + 63; m++)
                    C_l[k -128*P] [-32 * i + l] +=
                        B_l[-64*j + m] [-32*i + l] *
                        A_l[k -128*P] [-64*j + m];
        // write back a submatrix of C
        put C_l[k][l] to C[k + 128*P] [32*i + l]          // PUT C
            for 0 <= k <= 127, 0 <= l <= 31 tag 1;
        wait 1;
    }
}
```

In the example above, all of the multiplications associated with multiplying the 1024×1024 matrix A with another 1024×1024 matrix B, i.e., a total of 1024×1024×1024 multiplications, are distributed to eight processors, each indexed by a unique value of P in the range 0 through 7. Each of the eight processors receives in one iteration subsets of matrices A and B from the host memory, computes a corresponding subset of the product matrix C, and transfers the computer subset to the host memory. In this embodiment, a local matrix A_I, that corresponds to the matrix A and that is stored in a local memory is a 128×64 matrix, and a local matrix B_I, corresponding to the matrix B and stored in the local memory is a 64×32 matrix. As such, the local product matrix C_I, corresponding to matrix C and stored in the local memory, is a 128×32 matrix. Thus, in each iteration each of the eight processors can compute a 128×32 subset of the product matrix C. As each of the eight processors can compute a different 128×32 subset of matrix C in one iteration, in a total of 32 iterations the eight processors can compute the entire 1024×1024 matrix. These 32 iterations are represented by LOOP i.

In each iteration of LOOP i, the statement identified by GET B and the subsequent "for" iterator retrieve a subset of the matrix B to the corresponding local memory B_I of each of the eight processors. Similarly, the statement identified by GET A and the corresponding "for" iterator retrieve a subset of the matrix A to the corresponding local memory A_I of each of the eight processors. Finally, the statement identified by GET C and the corresponding "for" iterator retrieve a subset of the matrix C, which is initialized to zero, to the corresponding local memory C_I of each of the eight processors. After the computation of the product of the retrieved subsets, as indicated by the statements labeled as kernel, the PUT C statement and the corresponding "for" iterator transfer the computed subset C_I from the local memories of the respective processors to the matrix C in the global memory.

With reference to the example above, x is a vector that represents values of loop iterators that scan element-wise transfers. They are the vector (k, l) in the example above. As described above, y is a vector of loop counters and other expressions whose value stay constant during the transfer. In the foregoing example, they are the vector (i, j). $\mathcal{D}$ is a set of constraints that define the values taken by x as a function of y. In some cases, x takes on values independently of y. $\mathcal{D}$ in the foregoing example includes the constraints: $0 \le k \le 63$, $0 \le l \le 31$; these constraints do not depend on y (i.e., (i,j)). The functions f and g define which data element of A and B participates in the transfer, for each value of i, j, k, and l.

It should be understood that the example above is only a representative of memory transfers. Memory transfers, in general, may be required according to various computations, such as factorization, decomposition, solutions of linear and non-linear equations, inverse, transpose, and/or conjugate computations, etc. Also, such operations may be performed not only on two-dimensional matrices, but on scalar data, arrays, multi-dimensional arrays or tensors, and other simple or complex data structures. In general, in performing a specified operation/computation on a given set of operands using two or more processors, it is necessary to transfer at least portions of one or more operands stored in a global or master memory to one or more memories. These memories may be distributed, and may be associated with the different processors to which at least parts of the computation are designated. A memory associated with a processor is often called a local memory. At least portions of partial and/or final results may also be transferred from one or more memories, e.g., local memories, to the master or global memory.

In the distributed matrix multiplication example above, the generic "get" and "put" operations represent transfer of individual data elements between two memories, e.g., a global memory and one or more local memories. In some computations, a transfer can occur between two local memories. The "for" iterators, together with the corresponding "get" and/or "put" operations represent a transfer of a set of data elements. As described above in connection with local memory compaction, such transfer can include a contiguous block of data elements and/or a set of data elements spaced in a certain deterministic manner, such as evenly spaced, evenly spaced, i.e., strided blocks of contiguous data elements, etc. As described below, in various embodiments a data transfer operation is configured such that a data required for at least a part of a computation and/or produced by at least a partial computation can be transferred from one memory to another. The data transfer operation so configured can transfer, without requiring explicit specification of any iterators, data that may be contiguous or spaced according to a deterministic pattern in one or both memories.

The data transfer operation optimization component of the mapper, such as a polyhedral mapper, can turn the generic communication operations into an optimized set of data transfer operations (e.g., DMA transfer commands) supported by the underlying target architecture. Recall that the generic communication commands may have the forms:

get A[f(x, y)] from B[g(x, y)] for x$\in \mathcal{D}(y)$ tag t
put A[g(x,y)] to B[f(x,y)] for x$\in$D(y) tag t
wait t

Here, get and put operations are issued to transfer an arbitrary region of memory from one memory to another. All operations are tagged with an integer tag, and can be blocked until completion when a wait operation with the corresponding tag is executed.

While a wide array of different data transfer architectures is available, the following list of characteristics typically hold for all of them:

- All data transfer operations can be asynchronously issued, i.e., the data transfer operations can be executed without delaying computation code, thus allowing communication and computation to overlap in time.
- Multiple data transfer operations may be issued in parallel, up to some limit.
- One-dimensional strided accesses are possible.
- While many data transfer engines impose size and alignment restrictions on the transfer parameters, the data structures allocated by the compiler and by the mapper generally can be made naturally aligned. Thus part of the alignment restrictions can be trivially satisfied at compile time.

In some embodiments, the above data transfer operations are mapped into one or more of the following 1-D strided DMA primitives:

```
dma_get{src=s, dst=t,
count=n, bytes=b,
srcStride=ss,
dstStride=ds,
tag=tag}
dma_put{src=s,dst=t,
count=n, bytes=b,
srcStride=ss,
dstStride=ds,
tag=tag}
dma_wait{tag=tag}
```

where the semantics of dma_get in terms of pseudo code is:

```
dma_get{src=s,dst=t,
        count=n,bytes=b,
        srcStride=ss,
        dstStride=ds,
        tag=tag} =
for (i = 0; i < count: i++, s += ss, d += ds) {
    memcpy(d, s, b);
}
```

The semantics of dma_put can be stated similarly as that for dma_get.

Typically, in terms of execution cost of data transfer operations: (i) there is a substantial startup cost for each transfer operation, and (ii) there is a transfer cost for each byte in the message. The actual startup cost and transfer rate can be parameterized in the machine model of the target machine. However, given such constraints, the data transfer operation optimization problem can be formulated as: (A) Minimize the total number of data transfer commands issued; and/or (B) Minimize the total number of bytes transferred.

Example

Consider the following communication operations/commands embedded in a loop nest:

```
double A[128][128];
double B[128][128];
double A_l[16][16];
double B_l[16][16];
double A_l[16][16];
for (i = ...) {
    for (j = ...) {
        ...
        get A_l[k][l] from A[16 * P + k][16 * i + l]
            for 0 <= k <= 15, 0 <= l <= 15 tag 0;
        get B_l[k] from B[16 * j + k][16 * + k]
            for 0 <= k <= 15 tag 0;
        get C_l[k][l] from C[16 * P + k][16 * i + l];
            for 0 <= k <= 15, 0 <= l <= 15 tag 0;
        wait 0;
        ...
    }
}
```

In each iteration of i and j, a rectangular 16×16 block of A is fetched into A_1, a diagonal segment of B is fetched into B_1, and another rectangular 16×16 block of C is fetched into C_1, but with its rows and columns permuted, i.e., a corner turn.

For these operations, one possible mapping, using DMA operations for data transfers, is as follows:

```
for (i = ...) {
  for (j = ...) {
    ...
    dma_get{src=&A[16*P][16*i], dst=&A_l[0][0],
            count=16, bytes=16*sizeof(double),
            srcStride=128*sizeof(double),
            dstStride=16*sizeof(double), tag=0};
    dma_get{src=&B[16*j][16*i], dst=&B_l[0],
            count=16, bytes=sizeof(double),
            srcStride*128*sizeof(double),
            dstStride=sizeof(double), tag=0};
    for (l = 0; l < 15; l++) {
        dma_get{src=&C[16*P][16*i+1],
                dst=&C_l[l][0],
                count=16,
                bytes=sizeof(double),
                srcStride=128*sizeof(double),
                dstStride=sizeof(double),
                tag=0
            };
    }
    dma_wait{tag=0};
    ...
  }
}
```

The DMA mappings are generated as follows: For array A, a strided DMA command is used to transfer 16 row portions of A, with each block having 16 elements. The result is compacted into the array A_I. For array B, a strided DMA command can be used to transfer a diagonal segment of B into B_I. For example, a source stride of 128*sizeof (double) corresponds to the transfer of a column, i.e., the transfer of B[16*j*+k][16*i*]. A source stride of 129*sizeof (double) can transfer B[16*j*+k][16*i*+k], i.e., the diagonal elements of B. The strided DMA command transfers 16 blocks each of one element long. The source stride is staggered so that only the diagonal elements are transferred. For array C, 16 strided DMA operations are issued to transfer C_I. In this case, a single strided DMA operation is insufficient because it is a corner turn. Finally, the dma_wait command is issued to block subsequent operations until all DMA operations (data transfer operations, in general) have completed. In some embodiments, it is possible to overlap communication and computation by delaying the wait operation further.

In the example above, one data transfer operation corresponding to the first dma_get command would transfer 16 units of data, as specified by "count=16." Each unit would include 16 double-precision values, as specified by "bytes=16*sizeof(double)," and each unit will be strided by 128*sizeof(double), as specified by the parameter "srcStride." Thus, a data transfer operation per the first dma_get command will transfer 256 double-precision values in a single iteration.

With respect to the second dma_get command, a single data transfer operation per that command would also transfer 16 units of data, as specified by "count=16." Each unit in this operation, however, would include only one double-precision value, as specified by "bytes=sizeof(double)," and each unit/chunk will be strided by 128*sizeof(double), as specified by the corresponding "srcStride." Thus, a data transfer operation per the second dma_get command will transfer 16 double-precision values in a single iteration.

With respect to the first dma_get command, which transfers data from the matrix A to matrix A_I, the data transfer size corresponding to one data transfer operation of that dma_get command is 256*sizeof(double), which is no greater than the size of the matrix A_I. In this case, the data transfer size corresponding to one data transfer operation is actually the same as the size of A_I, so only one iteration of this data transfer operation is required.

The parameter "count" can be changed to a different value, e.g., 8, and accordingly, the data transfer size corresponding to one data transfer operation of the first dma_get command as modified would be 128*sizeof(double), which is less than the size of the matrix A_I. In this case, two iterations of the data transfer operation would be required. In the exemplary dma_get and dma_put commands, the param-

eters "count" and "bytes" together determine the data transfer size corresponding to one data transfer operation. The values of either or both of these parameters can be determined according to various processor parameters (e.g. word-length), memory parameters, and the parameters of communication links therebetween. In general, the data transfer size corresponding to one data transfer operation can be selected based on one or more of these system parameters, directly or via one or more parameters of the transfer operation/command.

Optimization of Data Transfer Operations

In some embodiments, the DMA optimization algorithm in the mapper uses a C style storage layout order, i.e., row major order. When compiling a language with a different storage layout order, suitable modification can be made to the parameterizable machine model. In some embodiments, each communication command is optimized separately. Given a communication command:

get/put A[ƒ(x,y)] from/to B[g(x, y)] for x∈$\mathcal{D}$(y) tag t

$\mathcal{D}$(y) is generally defined as a (d+p)-dimensional union of polyhedra, where p is the number immutable loop dimensions y, and d is the number of loop dimensions x that can be grouped into bulk communications. The order in which the iterations of the loops that correspond to immutable dimensions is not altered. When D(y) is a union of polyhedra, each polyhedron in the union can be optimized independently. The decomposition of D(y) into polyhedra is not unique. In one embodiment, the decomposition of D(y) results in a minimal number of polyhedra. In another embodiment, the decomposition results in a disjoint set of polyhedra, i.e., a minimal set of transferred elements. Hence transfers where D(y) is a polyhedron are considered. Every d-dimensional integer point x corresponds to the transfer of a single element of array A to an element of array B. From the description of the transfer as given by the transfer command, a set of c nested loops that execute the transfer in bulk is generated. In order to determine the order in which the bulk transfers in D(y) will occur, an ordered set $S=(s_1, \ldots, s_{(d-c)})$ of (d−c) linearly independent vectors is defined.

Let x_0 be a particular value of x in D(y). Let $M(x_{0,y})$ be a subset of the transfers of D(y) defined by:

$M(x_0,y)=\mathcal{D}(y)\cap S(x_0,y),x_0\in\mathcal{D}(y)\cap Z^{d+p+1}$

where $S(x_0, y)$ is the (d−c) dimensional subspace of $\mathbb{Q}^d$ that is spanned by a set of (d−c) vectors $S=(s_1, \ldots, s_{(d-c)})$, and contains point x_o. It is always possible to find an integer unimodular matrix V whose rightmost column vectors span $S\,\mathbb{Q}^{(d-c)}$ (proof uses Hermite normal form). Let $\mathcal{D}'$ be the image of $\mathcal{D}$ by V^{-1}, the inverse matrix of V. The (d−c) last variables of $\mathcal{D}'$ scan the image of the M's. Within a data set to be communicated, there is no legality constraint on the order in which the data should be communicated, so any unimodular V is legal.

Let $M'(x'_0, y)$ be the image of $M(x_0, y)$ through V^{-1}. Each distinct $M'(x'_0, y)$ is meant to be communicated at once by a single strided communication operation. Let $\alpha(w)=\alpha\cdot w+\alpha$ and $b(z)=b\cdot z+\beta$ be the storage functions for arrays A and B, where a and b are integer vectors, α is the base address of array A, β is the base address of array B, and w and z are the coordinates of an array element of A and B respectively. The exact subset of addresses in array A that are accessed in the data transfer defined by $M'(x_0', y)$ The exact data set M' can be communicated at once using a communication operation if

$m_a(x_1', \ldots, x_c')=\{(a_{c+1}, \ldots, a_d)\cdot(x_{c+1}', \ldots, x_d')^T | x'\in D'(y)\}$

and

$m_b(x_1', \ldots, x_c')=\{(b_{c+1}, \ldots, b_d)\cdot(x_{c+1}', \ldots, x_d')^T | x'\in D'(y)\}$

are each defined by exactly one $\mathbb{Z}$-polyhedron for $x'\in\mathcal{D}'$. This condition, which is sufficient but not necessary, is always true in the case when (d−c)=1 and ƒ(x,y) and g(x,y) are invertible. Hence, a heuristic for minimizing the number of communication operations is to find S that minimizes c. This would not take into account the number of data along S but only the number of data dimensions in each communication operation.

In general, m_a is communicated onto m_b for all the valid integer values of $(x'_1, \ldots, x'_{c-1})$, in $\mathcal{D}'$. This set of valid values is basically the projection of $\mathcal{D}'$ on the subspace of variables $(x'_1, \ldots, x'_{c-1})$, which defines the iteration domain of the communication operation to be derived. If m_a and m_b are each one $\mathbb{Z}$-polyhedron, a base address and stride is computed for both sides as:

$base(A,(x_1', \ldots, x_c')^T)=(a_1, \ldots, a_c)(x_1', \ldots, x_c')^T)+\min(m_a(x_1, \ldots, x_c))+\alpha$

$base(B,(x_1', \ldots, x_c')^T)=(b_1, \ldots, b_c)(x_1', \ldots, x_c')^T)+\min(m_b(x_1, \ldots, x_c))+\beta$

The strides are given by the linear part of the supporting lattices of m_a and m_b, which are constant across the values of y and $(x_1, \ldots, x_c)^T$ (up to the "center" of the lattice, which is not helpful with respect to strides and is accounted for in the base address).

The number of packets in the message is given by the Ehrhart polynomials of m_a and m_b. If there is a bijection between m_a and m_b, this number is the same. If not, it means that there are gather or scatter operations or both, and the number of transferred data may be ill-defined. If there is either a scatter or a gather involved, the number of packets is the maximum between the Ehrhart polynomials of m_a and m_b. In some embodiments, instead of computing and comparing the Ehrhart polynomial explicitly, the Ehrhart polynomial of the reference that has the highest rank can be taken. Generally, the size of a packet is set to the size of the array element (assumed to be the same on both sides).

Special Cases

Big packets: When the size of m_a and m_b is independent of $(x_1, \ldots, x_{c-1})$, and when they are both contiguous (their supporting lattice is $\mathbb{Z}$), each m_a and m_b can be considered as one packet and further data dimensions can be aggregated into the communication operation. In this case, the same process is performed on the iteration domain, by considering what is currently a message as a data element.

Strides not allowed on one side: When the communication layer used by the communication commands being generated does not allow strided communication on one side, a desirable option is to build messages made of contiguous data on that side. Without loss of generality, it is assumed that the side for which strided communication is not allowed is the side of array A. Hence, the column-vectors of S are chosen such that the last dimension of $ƒV^{-1}$ are the (d−c) last canonical vectors. For performance considerations, communicating contiguous data on one side is usually a good choice, even when there is no restriction on the communication layer used.

Strides not allowed on any sides: When strides are not allowed on both sides, unless there exists c and S such that the rightmost column-vectors of both $ƒV^{-1}$ and gV^{-1} are the last canonical column-vectors, direct transfer is impossible. In this case, it is necessary to copy output data into a buffer with the right data layout before sending it and/or after receiving it. This buffered communication mode is called copy-and-communicate. As the amount of data to communicate is usually minimized, the sending side may copy its output data into a buffer, and the buffer is sent (and received) as a whole. The data is laid out as needed by the recipient

(which is computed by the local memory optimization component). The generic communication operations are turned into copies to the local buffer, and a buffer_send( ) is issued. Normally, the recipient does not need to modify the layout of the incoming data, as it was laid out optimally for it by the sender.

Bijection between both sides: When the elements to be communicated between A and B are related by a bijection (one-to-one relation) it is easier to work on one of the images, i.e., $f(\mathcal{D}(y),y)$ or $g(\mathcal{D}(y),y)$. They are generally defined by a union of $\mathbb{Z}$-polyhedra, so it is possible to form communication commands with (d−c)=1 for each element of this union.

Simplifying the data transfers by transferring more: In some cases, communicating more data than necessary allows for producing simpler communication operations. It is important, however, to not update a value invalidly, which would break the program's semantics. Examples where enlarging the data set seems profitable include: interleaved messages whose union is a $\mathbb{Z}$-polyhedron; data sets with small data holes; and when the data set to be communicated can be overapproximated by a bounding hyper-rectangular box, and when the density of data in the box is high enough. The increase in the data to be communicated is a selectable system parameter that can be set to 50%, 60%, 75%, etc.

Optimizing for data transfer size: Some communication engines (e.g., DMA hardware engines) reach optimal performance when the message has a certain size. Also, certain communication libraries do not allow message sizes to exceed a given size. In either case, tiling can be applied to $\mathcal{D}'$, in such a way that the number of data in the tile equal the optimal or maximal number of data. Typically, if the trip count of the innermost loop level is too short, more tiling levels may be used. In general, because the data can be sent in any order, tiling copy operations is generally legal. The m_a and m_b are then formed with the "intra-tile" loops.

Optimizing for memory banks: Many hardware memories are organized in banks. Data transfers are generally faster when successive data transfers are issued to different memory banks. Hence, in addition to forming messages with certain properties, it is desirable to schedule the communication operations in such a way that consecutive operations are issued on different memory banks. For this, the stride e at loop level c must be such that:

$$k*\text{bank_size}*nb_\text{banks}+\text{bank_size}\leq e\leq(k+1)*(\text{bank_size}*nb_\text{banks})-\text{bank_size}$$

where k is existentially quantified. As the full access function is defined by $\alpha(f(Vx', y)$, i.e., $a.f.Vx'$, where x_c' is incremented the corresponding stride in A is given by the c^{th} columnvector of a f V. Formally, the constraint is then:

$e=(af V\cdot_c)$

$\exists k$: $(k\times\text{bank_size}\times(\text{nb_banks}+1))\leq e\leq((k+1)\times\text{bank_size}\times\text{nb_banks})$

Other constraints may also be taken into consideration. For example, V has to be full-rank. In particular, V it has to be independent of the subspace S spanning the message. Also, V has to be unimodular and its last (d−c) columnvectors must span S.

A constructive solution technique is employed to build a set of solutions to satisfy all of these constraints. In some embodiments, first k can be eliminated by using the maximum size of a message and computing k_{min}, the minimum k that makes $((k\times\text{bank_size}\times(\text{nb_banks}+1))$ greater than the maximum transfer data set size supported by the underlying architecture. Then, let T defined by V=(T|S). A new $v._c$, denoted as $v'._c$. is defined as a combination of the vectors of V such that the resulting matrix, say V', is still unimodular. In other words, the relation V'=VU must hold, where U is integer unimodular. As the other column-vectors of V are not changed, U may be of the form:

$$U=\begin{pmatrix}1&0&\dots&u_{1,c}&\dots&0&0\\0&1&\dots&\vdots&\dots&0&0\\\vdots&&\ddots&\vdots&&&\vdots\\\vdots&&&u_{c,c}&&\vdots&\\\vdots&&&\vdots&\ddots&&\vdots\\0&0&\dots&\vdots&\dots&1&0\\0&0&\dots&u_{d,c}&\dots&0&1\end{pmatrix}$$

Unimodularity of U can be ensured by setting $u_{c,c}=1$. Thus, the problem becomes: build a linear combination of the vectors of V such that:

$$k_{min}*\text{bank_size}*nb_\text{banks}+\text{bank_size}\leq af Vu._c\leq(k_{min}+1)*(\text{bank_size}*nb_\text{banks})-\text{bank_size}$$

with $u_{c,c}=1$, which is a system of constraints with (d−1) free variables. Any solution is valid, even though to get clean code the coefficients of $u._c$. may be minimized. In some embodiments, the stride e may be set to a fixed value such as bank_size to make the constraint set simpler. This technique can find solutions if and only if the data set to be communicated spans more than one memory bank.

Interleaving data transfers for different communication operations: In some embodiments transfers of data that belong to different memory banks may be interleaved. This can be done by first computing the communication operations and their iteration domain, and then fusing loops of the same dimension and, if possible, of "similar" shapes, in order to limit the amount of control overhead that such fusion would entail.

Illustrated in FIG. **9** are computing apparatus and computer software products consistent with provided embodiments. Computing apparatus **720** includes processor **660**, memory **670**, storage medium **680**, and in some embodiments input port **690** and network interface **710**. In many provided embodiments, storage medium **680** contains a set of processor executable instructions that when executed by processor **660** configure computing apparatus **720** to implement the modules and methods described herein. In one embodiment, storage medium **680**, containing the set of processor executable instructions resides in another computing apparatus **720** across network **730**. In an embodiment of a computer software product, computer software product **700** is a computer readable storage medium containing processor executable instructions sufficient that when executed by processor **660** configure computing apparatus **720** to implement the above described modules and methods. Further, computer software product, in some embodiments consists of a physical medium configured to interface with input port **690** to allow its contents to be copied to storage medium **680**. In other embodiments, computer software product **700** is an internal storage medium, such as **680**. An additional embodiment of computing apparatus **720** includes a plurality of processors **680**(*a-n*), a plurality of memories **670**(*a-n*), a storage medium **680** and in some embodiments input port **690** and network connection **710**. In some embodiments, one or more processors **680**(*a-n*) is a host, while others are modeled in the form of a grid.

Thus, it is seen that methods apparatus and computer software products for allocating arrays in memories with

constrained memory requirements according to the way those arrays are accessed is provided. Specifically, methods and an apparatus for optimizing source code on a custom first computing apparatus for execution on a second computing apparatus, whereby data communications on the second computing apparatus are minimized are provided. One skilled in the art will appreciate that the present invention can be practiced by other than the above-described embodiments, which are presented in this description for purposes of illustration and not of limitation. The specification and drawings are not intended to limit the exclusionary scope of this patent document. It is noted that various equivalents for the particular embodiments discussed in this description may practice the invention as well. That is, while the present invention has been described in conjunction with specific embodiments, it is evident that many alternatives, modifications, permutations and variations will become apparent to those of ordinary skill in the art in light of the foregoing description. Accordingly, it is intended that the present invention embrace all such alternatives, modifications and variations as fall within the scope of the appended claims. The fact that a product, process or method exhibits differences from one or more of the above-described exemplary embodiments does not mean that the product or process is outside the scope (literal scope and/or other legally-recognized scope) of the following claims.

What is claimed is:

1. A method of facilitating exchange of data between a first memory and a second memory, the method comprising:

receiving a mapping between a global data structure maintained in the first memory and a local data structure maintained in the second memory, the second memory being associated with a particular processor;

deriving an asynchronous bulk communication operation for transferring data between the global and local data structures, the asynchronous bulk communication operation comprising (i) access to the global data structure based on, at least in part, a structure one or more characteristics of the global data structure, and (ii) access to the local data structure based on, at least in part, one or more characteristics of the local data structure, for minimizing a parameter of a total data transfer between the first and second memories;

wherein a characteristic of the global data structure or a characteristic the local data structure comprises at least one dimension and a sequence of indices corresponding to each dimension; and

wherein the parameter comprises at least one of a number of data transfer operations required for the total data transfer, and a size of data to be transferred for the total data transfer.

2. The method of claim **1**, wherein the local data structure is based on, at least in part, a local memory compaction corresponding to a portion of a specified computation, the portion being designated to the particular processor.

3. The method of claim **2**, wherein the local memory compaction corresponds to a minimum total data size, the method further comprising selecting the local data structure such that:

a size of the local data structure is greater than the minimum total data size; and

at least one of: (i) the access to the global data structure comprises a first stride of a first constant value, and (ii) the access to the local data structure comprises a second stride of a second constant value.

4. The method of claim **1**, wherein data in the local data structure is at least one of processed or to be processed by the particular processor.

5. The method of claim **1**, wherein:

the parameter comprises a number of data transfer operations required for the total data transfer;

a data transfer size corresponding to one data transfer operation is no greater than the size of data to be transferred for the total data transfer; and

at least one of selecting the access to the global data structure or the access to the local data structure comprises selecting the data transfer size that minimizes the number of data transfer operations.

6. The method of claim **5**, further comprising selecting the data transfer size corresponding to one data transfer operation based on, at least in part, a capacity of a communication link between the first and second memories.

7. The method of claim **1**, wherein:

at least one of: (i) the first memory comprises a first set of memory banks, or (ii) the second memory comprises a second set of memory banks;

at least one of: (i) the access to the global data structure comprises a first stride of a first constant value, or (ii) the access to the local data structure comprises a second stride of a second constant value; and

at least one of: (i) selecting the access to the global data structure comprises selecting, based on at least in part, a number of banks in the first set and a size of a memory bank in the first set, the first stride such that consecutive data transfer operations correspond to distinct memory banks of the first set, or (ii) selecting the access to the local data structure comprises selecting, based on at least in part, a number of banks in the second set and a size of a memory bank in the second set, the second stride such that consecutive data transfer operations correspond to distinct memory banks of the second set.

8. The method of claim **1**, wherein:

the global data structure comprises at least one dimension;

the local data structure comprises at least one dimension; and

at least one dimension of the local data structure is smaller than a corresponding dimension of the global data structure.

9. The method of claim **1**, wherein:

the local data structure comprises at least one set of data elements; and

deriving the asynchronous bulk communication operation comprises computing: (i) a count representing a number of sets of data elements; and (ii) a size of each set of data elements.

10. The method of claim **9**, wherein computing the size of a set of data elements comprises determining a size of a data element.

11. The method of claim **9**, wherein computing at least one of the count or the size of a set of data elements is based on, at least in part, a capacity of a communication link between the first and second memories.

12. The method of claim **1**, wherein:

the local data structure comprises a plurality of sets of data elements;

the global data structure comprises a plurality of corresponding sets of data elements; and

deriving the asynchronous bulk communication operation comprises computing a stride representing a distance between a first set of data elements in the global data structure and a second set of data elements in the global data structure.

13. The method of claim 12, wherein computing the stride is based on, at least in part, a dimension of the global data structure.

14. The method of claim 1, wherein:

the local data structure comprises a plurality of sets of data elements;

the global data structure comprises a plurality of corresponding sets of data elements; and

deriving the asynchronous bulk communication operation comprises computing a stride representing a distance between a first set of data elements in the local data structure and a second set of data elements in the local data structure.

15. The method of claim 14, wherein computing the stride is based on, at least in part, a dimension of the local data structure.

16. The method of claim 1, wherein deriving the asynchronous bulk communication operation comprises determining at least one of: (i) a starting address in the first memory, and (ii) a starting address in the second memory.

17. The method of claim 1, wherein

the global data structure comprises a first dimension and a second dimension;

the local data structure also comprises the first dimension and the second dimension; and

deriving the asynchronous bulk communication operation comprises generating: (i) a start address in the first memory, based on, at least in part, the first dimension, and (ii) a start address in the second memory, based on, at least in part, the second dimension, such that a set of data elements in the global data structure and a corresponding set of data elements in the local data structure are arranged in the first and second dimensions, respectively.

18. The method of claim 1, further comprising transferring at least a portion of data from the first memory to the second memory according to the asynchronous bulk communication operation, thereby storing the data in the second memory.

19. The method of claim 1, further comprising transferring data from the second memory to the first memory according to the asynchronous bulk communication operation, thereby storing at least a portion of the data in the first memory.

20. The method of claim 1, wherein the step of deriving the asynchronous bulk communication operation is designated to a computer distinct from the processor.

21. The method of claim 20, wherein the computer is configured to derive a sequence of asynchronous bulk communication operations, the sequence comprising the asynchronous bulk communication operation, and each operation in the sequence corresponding to a loop index and being further based on, at least in part, the loop index.

22. The method of claim 20, wherein the computer is configured to derive a sequence of asynchronous bulk communication operations, the sequence comprising the asynchronous bulk communication operation, and each operation in the sequence: (i) corresponding to a distinct processor specified by a processor index, and (ii) being further based on, at least in part, the processor index.

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